

Global Mathematical Object Associated with Papers I–V

Exact resolution, Omega arithmetic, floor dynamics, and radial icosahedral harmonics

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SYNTHESIS SPECIFICATION

Scale-Resolved State Dynamics (SRSD) remains the sole frozen foundation. This document does not introduce a second foundation, a universal physical constant, or a new claim of priority. It forms one explicit multi-sorted mathematical object from the results of Papers I–V and proves that the stated interfaces are well typed and exact. Arithmetic, dynamics, geometry, harmonic analysis, and finite computation retain their separate hypotheses.

Abstract

Five previously specified bodies of mathematics are assembled into one concrete object. The frozen SRSD encoder–decoder family supplies the exact coordinate layer. The field generated by $\Omega_0 = \phi + \pi$ supplies the exact arithmetic layer and the common parameters ϕ and $\lambda = \phi/\Omega_0$. The real floor-quantized contraction and the rational floor map supply two dynamical layers. Their quotient trajectories agree exactly on the complete unsigned 15-bit domain, while their different residuals remain explicitly distinguishable. The radial Class-I icosahedral family supplies a geometric-harmonic layer whose measures and moments transport losslessly through SRSD coordinates. Canonical symbolic records and ordered Euclidean ledgers supply finite reversible representations. The resulting object is a conservative assembly: each paper is recovered by a typed projection, and no cross-branch identity is asserted beyond the displayed gluing laws.

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1 Scope and construction rule

The goal is not to merge every subject into one undifferentiated theory. The goal is to specify one object with several sorts, each retaining its own domain, operations, and proof status, and then to state the exact maps that connect the sorts. In particular:

- (i) Paper III supplies the frozen quotient–residual foundation.
- (ii) Papers I and II supply dynamical specializations of the floor operation.
- (iii) Paper V supplies the exact Ω_0 arithmetic, positive-ray normalization, canonical symbolic records, and the finite 15-bit bridge.
- (iv) Paper IV supplies the independent radial icosahedral and harmonic branch.
- (v) Only the interfaces proved below are used to glue the branches.

The assembly is a multi-sorted tuple, not a claim that all component spaces have one common binary operation. Optional categorical or transported-algebra descriptions can be added later, but they are not part of the frozen foundation and are not needed here.

2 Notation and disambiguation

Two source papers use the letter τ for different quantities. The global object separates them permanently:

$$\tau_0 := \frac{1}{\Omega_0} \quad \text{is the fixed reciprocal of } \Omega_0, \quad h > 0 \quad \text{is an arbitrary quantizer spacing.}$$

The remaining shared constants are

$$\phi = \frac{1 + \sqrt{5}}{2}, \quad \Omega_0 = \phi + \pi, \quad \lambda = \frac{\phi}{\Omega_0} = \phi\tau_0, \quad \mu = 1 - \lambda = \frac{\pi}{\Omega_0}.$$

The rational 15-bit specialization is written

$$S_0 = 10^{15}, \quad Q_0 = 339949771344778, \quad q_0 = \frac{Q_0}{S_0}.$$

Thus a general SRSD scale vector is $\mathbf{s}$, a Paper-I lattice spacing is h , and the fixed Paper-II/Paper-V denominator is S_0 .

3 The five component structures

3.1 Frozen SRSD resolution

For $m \geq 1$ and $\mathbf{s} = (s_1, \dots, s_m) \in (0, \infty)^m$, put

$$\Delta_{\mathbf{s}} = \text{diag}(s_1, \dots, s_m), \quad \mathcal{C}_{\mathbf{s}} = \prod_{i=1}^m [0, s_i), \quad \mathcal{R}_{\mathbf{s}} = \mathbb{Z}^m \times \mathcal{C}_{\mathbf{s}}.$$

Define

$$q_{\mathbf{s}}(y)_i = \lfloor y_i / s_i \rfloor, \quad r_{\mathbf{s}}(y) = y - \Delta_{\mathbf{s}} q_{\mathbf{s}}(y), \\ E_{\mathbf{s}}(y) = (q_{\mathbf{s}}(y), r_{\mathbf{s}}(y)), \quad D_{\mathbf{s}}(d, \rho) = \Delta_{\mathbf{s}} d + \rho.$$

The frozen exact-resolution identities are

$$D_{\mathbf{s}} E_{\mathbf{s}} = \text{id}_{\mathbb{R}^m}, \quad E_{\mathbf{s}} D_{\mathbf{s}} = \text{id}_{\mathcal{R}_{\mathbf{s}}}. \quad (1)$$

For an observation map $H : X \rightarrow \mathbb{R}^m$, exact resolution preserves its fiber relation:

$$\text{FibKer}(E_{\mathbf{s}} \circ H) = \text{FibKer}(H). \quad (2)$$

Any later omission, rounding, compression, or projection is a separate record map and must be tested for injectivity on the reached image.

3.2 The Ω_0 arithmetic structure

Let

$$K_{\Omega} = \mathbb{Q}(\pi, \sqrt{5}) = \mathbb{Q}(\tau_0, \lambda).$$

Then $K_{\Omega}/\mathbb{Q}(\tau_0)$ is quadratic and

$$\lambda^2 - \tau_0 \lambda - \tau_0^2 = 0. \quad (3)$$

Every $x \in K_{\Omega}$ has a unique form

$$x = a + b\lambda, \quad a, b \in \mathbb{Q}(\tau_0). \quad (4)$$

The nonidentity conjugation σ_K over $\mathbb{Q}(\tau_0)$ satisfies

$$\sigma_K(\lambda) = \tau_0 - \lambda,$$

and, for $x = a + b\lambda$,

$$\mathrm{Tr}(x) = 2a + b\tau_0, \quad (5)$$

$$\mathrm{N}(x) = a^2 + ab\tau_0 - b^2\tau_0^2. \quad (6)$$

The field carries the canonical exact symbolic record of Paper V: normalized rational-function coefficients in the basis $(1, \lambda)$, followed by the fixed deterministic byte serialization. Denote this injective record map by

$$\mathrm{Can}_\Omega : K_\Omega \longrightarrow \mathrm{Byte}^*. \quad (7)$$

This stores a finite symbolic expression, not a terminating binary expansion of the represented transcendental real number.

For every odd prime p , this arithmetic sort also contains the checked cyclotomic intersections

$$K_\Omega \cap \mathbb{Q}(\zeta_p) = \begin{cases} \mathbb{Q}(\sqrt{5}), & p = 5, \\ \mathbb{Q}, & p \neq 5, \end{cases}$$

and therefore the prime Fourier kernel is the constant-vector line for $p \neq 5$, while $p = 5$ is the exceptional quadratic case.

3.3 The real quantized-contraction structure

For $0 < \alpha < 1$ and $h > 0$, define

$$Q_h(y) = h \lfloor y/h \rfloor, \quad T_{\alpha,h}(z) = Q_h(\alpha z).$$

The Paper-I Omega branch is the distinguished specialization $\alpha = \lambda$. Starting with $z_0 = x \in \mathbb{R}$, let

$$z_{k+1} = T_{\lambda,h}(z_k), \quad \varepsilon_k = \lambda z_k - z_{k+1} \in [0, h).$$

For $n \geq 1$,

$$\lambda^n x = z_n + \sum_{k=0}^{n-1} \lambda^{n-1-k} \varepsilon_k, \quad (8)$$

and

$$x = \lambda^{-n} z_n + \sum_{k=0}^{n-1} \lambda^{-k-1} \varepsilon_k. \quad (9)$$

Since $0 < \lambda < 1/2$, nonnegative states reach 0 and negative states reach $-h$ in finite time.

For the full $0 < \alpha < 1$ family, the same weighted decomposition and reverse formula hold with α in place of λ . Every nonnegative orbit reaches 0. Every negative orbit reaches a fixed state $-mh$, where the possible positive coefficients are exactly

$$m \in \left\{ 1, \dots, \left\lceil \frac{1}{1-\alpha} \right\rceil - 1 \right\}.$$

The affine form from Paper I is

$$A_\lambda(v) = 1 + \lambda(v - 1).$$

With $H(v) = v - 1$, one has

$$H \circ A_\lambda = M_\lambda \circ H, \quad M_\lambda(u) = \lambda u,$$

so $A_\lambda^n(v) = 1 + \lambda^n(v - 1)$.

3.4 The rational floor and ledger structure

For general integers $0 < Q < S$, put

$$F_{Q/S}(d) = \lfloor Qd/S \rfloor, \quad \rho_{Q,S}(d) = Qd - SF_{Q/S}(d).$$

For an orbit $d_{k+1} = F_{Q/S}(d_k)$, set

$$\rho_k = Qd_k - Sd_{k+1}, \quad L_N(d_0) = (d_N; \rho_0, \dots, \rho_{N-1}).$$

The exact reverse law and its closed form are

$$d_k = \frac{Sd_{k+1} + \rho_k}{Q}, \quad (10)$$

$$Q^N d_0 = S^N d_N + \sum_{k=0}^{N-1} \rho_k S^k Q^{N-1-k}. \quad (11)$$

If $d_k > 0$ for $k < N$, the same ordered record determines the exact maximal parameter interval preserving the full segment:

$$I(L_N) = \left[q - \min_{k < N} \frac{\rho_k}{Sd_k}, q + \min_{k < N} \frac{S - \rho_k}{Sd_k} \right) \cap (0, 1), \quad q = Q/S. \quad (12)$$

For completeness, the predecessor fiber of $y \in \mathbb{Z}_{\geq 0}$ is the finite half-open integer interval

$$F_{Q/S}^{-1}(y) = \left\{ \left\lceil \frac{S}{Q}y \right\rceil, \left\lceil \frac{S}{Q}y \right\rceil + 1, \dots, \left\lceil \frac{S}{Q}(y+1) \right\rceil - 1 \right\}.$$

The fixed finite subsystem uses $(Q, S) = (Q_0, S_0)$ on

$$D_{15} = \{0, 1, \dots, 32767\}.$$

Its floor map agrees exactly with $d \mapsto \lfloor \lambda d \rfloor$ on all of D_{15} , and every state reaches zero in at most ten steps.

3.5 The radial icosahedral harmonic structure

Let

$$V_{\text{ico}} = \{(0, \pm 1, \pm \phi), (\pm 1, \pm \phi, 0), (\pm \phi, 0, \pm 1)\}.$$

For frequency $f \geq 1$, let $X_f \subset \mathbb{S}^2$ be the complete radial Class-I grid and

$$|X_f| = 10f^2 + 2, \quad \mu_f = \frac{1}{|X_f|} \sum_{x \in X_f} \delta_x.$$

Let μ_{rad} be the radial pushforward of normalized planar face area, and define the invariant harmonic

$$I_6(x) = \sum_{v \in V_{\text{ico}}} P_6 \left(\frac{x \cdot v}{\sqrt{\phi + 2}} \right), \quad P_6(t) = \frac{231t^6 - 315t^4 + 105t^2 - 5}{16}.$$

Then

$$\mu_f \rightharpoonup \mu_{\text{rad}}, \quad (13)$$

$$X_f \text{ is an equal-weight spherical five-design,} \quad (14)$$

$$C_0 := \int_{\mathbb{S}^2} I_6 d\mu_{\text{rad}} > 0, \quad (15)$$

$$d_*(\mu_{\text{rad}}, \sigma_{\mathbb{S}^2}) = 6, \quad (16)$$

$$\mu_f(I_6) = C_0 + C_2 f^{-2} + C_4 f^{-4} + O(f^{-6}), \quad (17)$$

where the coefficients of f^{-1}, f^{-3}, f^{-5} vanish exactly and $C_0 > 0$ is established by the exact depth-six Bernstein certificate. More precisely,

$$0.130042106090748 < C_0 < 0.469495146930684.$$

Writing

$$\theta = \arctan(1/\phi), \quad V_5 = \frac{132}{25},$$

$$\mathcal{I} = \frac{693}{800}(5 - 3\phi)(7 + 5\theta),$$

$$B_{2,\text{face}} = \frac{239547}{320000} - \frac{180411}{320000}\phi + \frac{693}{2560}(1 - \phi)\theta,$$

the exact coefficients are

$$C_2 = \frac{V_5}{5} - \frac{\mathcal{I}}{2} - \frac{C_0}{5}, \quad C_4 = 2B_{2,\text{face}} - \frac{C_2}{5}.$$

4 Definition of the global object

Definition 4.1 (Global mathematical object). The global object associated with Papers I–V is the multi-sorted structure

$$\mathfrak{G} = (\mathfrak{S}_{\text{SRSD}}, \mathfrak{A}_\Omega, \mathfrak{D}_{\text{quant}}, \mathfrak{D}_{\text{rat}}, \mathfrak{H}_{\text{ico}}, \mathfrak{C}_{\text{fin}}; \mathcal{J}),$$

with the following sorts.

- (1) $\mathfrak{S}_{\text{SRSD}}$ is the complete frozen family $\{(\mathcal{R}_s, E_s, D_s)\}$ over all finite dimensions and positive scale vectors, together with the frozen fiber statements.
- (2) $\mathfrak{A}_\Omega$ is $(K_\Omega, \tau_0, \lambda, \mu, \sigma_K, \text{Tr}, \text{N}, \text{Can}_\Omega)$, including its normal forms and cyclotomic/Fourier specializations.
- (3) $\mathfrak{D}_{\text{quant}}$ is the family of real quantized systems $\{(\mathbb{R}, T_{\alpha,h}) : 0 < \alpha < 1, h > 0\}$ with their ordered residual ledgers and the distinguished Omega branch $\alpha = \lambda$.
- (4) $\mathfrak{D}_{\text{rat}}$ is the family of rational floor systems

$$\{(\mathbb{Z}_{\geq 0}, F_{Q/S}, \rho_{Q,S}) : 0 < Q < S, Q, S \in \mathbb{Z}\},$$

with exact reconstruction and parameter intervals, together with the distinguished finite subsystem (D_{15}, F_{q_0}) and its exact comparison with F_λ .

- (5) $\mathfrak{H}_{\text{ico}}$ is $(\{X_f, \mu_f\}_{f \geq 1}, \mu_{\text{rad}}, I_6, C_0, C_2, C_4)$.
- (6) $\mathfrak{C}_{\text{fin}}$ is the canonical symbolic-record map, the 15-bit byte-folding map, and the per-limb ordered witness ledgers.

The gluing data $\mathcal{J}$ consist exactly of equations (18)–(25) below and no unstated identifications.

The object is concrete: every sort is a set, field, family of maps, measure, or finite record system already realized in ordinary mathematics. The semicolon before $\mathcal{J}$ separates component data from cross-component laws.

5 The gluing laws

5.1 Paper I as an exact SRSD step

For scalar scale h , apply the one-dimensional encoder to λz_k . Because $z_{k+1} = h \lfloor \lambda z_k / h \rfloor$,

$$\boxed{E_h(\lambda z_k) = \left(\frac{z_{k+1}}{h}, \varepsilon_k \right), \quad D_h \left(\frac{z_{k+1}}{h}, \varepsilon_k \right) = \lambda z_k.} \quad (18)$$

Thus the Paper-I remainder is precisely the SRSD within-cell residual of the contracted value. The feedback rule selecting z_{k+1} from the quotient is application data; it is not supplied by the SRSD foundation.

5.2 Paper II as an exact SRSD step

Apply the one-dimensional encoder at scale S to the integer Qd_k . The Euclidean identity gives

$$\boxed{E_S(Qd_k) = (d_{k+1}, \rho_k), \quad D_S(d_{k+1}, \rho_k) = Qd_k.} \quad (19)$$

Accordingly, the ordered Paper-II ledger is a time-ordered sequence of exact SRSD coordinates for the values Qd_k , with the scale and update law retained.

5.3 Exact 15-bit quotient bridge and residual comparison

For $d \in D_{15}$, Paper V proves

$$\boxed{F_\lambda(d) = \lfloor \lambda d \rfloor = \lfloor q_0 d \rfloor = F_{q_0}(d).} \quad (20)$$

Since both maps preserve D_{15} , their complete forward trajectories agree there. If the common quotient at a step is d_{k+1} , define

$$\varepsilon_k^\lambda = \lambda d_k - d_{k+1}, \quad \varepsilon_k^q = q_0 d_k - d_{k+1} = \rho_k / S_0.$$

The two residuals are not identified. Their exact relation is

$$\boxed{\varepsilon_k^\lambda = \frac{\rho_k}{S_0} + (\lambda - q_0)d_k.} \quad (21)$$

The quotient bridge is therefore exact without asserting $\lambda = q_0$. Indeed, the first one-step disagreement occurs at $d = 42,189,065$, outside the declared 15-bit domain.

5.4 The positive-ray interface

For $m \geq 2$, let

$$C_m = \mathbb{R}_{>0}^m, \quad T(z) = \sum_{i=1}^m z_i, \quad p(z) = \frac{z}{T(z)}.$$

The cone–simplex map

$$\Psi(z) = (T(z), p(z))$$

is a real-analytic diffeomorphism

$$C_m \cong \mathbb{R}_{>0} \times \left\{ u \in \mathbb{R}_{>0}^m : \sum_i u_i = 1 \right\},$$

and the fibers of p are exactly the positive rays. In particular, every positively homogeneous map of degree k has the unique form

$$F(z) = T(z)^k f(p(z))$$

for its restriction f to the open simplex.

For $(x, y) \in \mathbb{R}_{>0}^2$, put

$$\Theta(x, y) = \left(\frac{x+y}{\Omega_0}, \frac{x}{x+y} \right).$$

Then

$$\boxed{\Theta(c\phi, c\pi) = (c, \lambda), \quad c > 0.} \quad (22)$$

Equivalently, $x/(x+y) = \lambda$ exactly when $(x, y) = c(\phi, \pi)$ for a unique $c = (x+y)/\Omega_0$. Combining this with the frozen SRSD injection gives, for every positive two-coordinate scale $\mathbf{s}$,

$$\boxed{E_s \left(\frac{(x, y)}{x+y} \right) = E_s(\lambda, \mu) \iff (x, y) = c(\phi, \pi) \text{ for a unique } c > 0.} \quad (23)$$

5.5 The radial geometric interface

For $\mathbf{s} \in (0, \infty)^3$, define the resolved pushforward measures

$$\widehat{\mu}_{f,\mathbf{s}} = (E_{\mathbf{s}})_{\#} \mu_f, \quad \widehat{\mu}_{\text{rad},\mathbf{s}} = (E_{\mathbf{s}})_{\#} \mu_{\text{rad}}. \quad (24)$$

For every integrable observable Y on $\mathbb{S}^2$, extend it arbitrarily off the sphere if necessary; on the supports in question, exact decoding gives

$$\boxed{\int_{\mathcal{R}_{\mathbf{s}}} Y(D_{\mathbf{s}}e) d\widehat{\mu}_{f,\mathbf{s}}(e) = \int_{\mathbb{S}^2} Y(x) d\mu_f(x),} \quad (25)$$

and the identical formula holds for μ_{rad} . No approximation is introduced by the resolution coordinates.

6 Existence and compatibility theorem

Theorem 6.1 (Existence and conservative compatibility). *The structure $\mathfrak{G}$ in Definition 4.1 exists. Its gluing laws are mutually compatible and have the following consequences.*

- (i) *Every Paper-I and Paper-II step is an exact instance of the frozen SRSD encoder–decoder identities, with the dynamics retained as separate application data.*

- (ii) *On D_{15} , the Paper-I unit-spacing quotient trajectory and the Paper-II/Paper-V rational quotient trajectory coincide, while Equation (21) retains their exact residual difference.*
- (iii) *The ordered rational remainder record reconstructs its orbit and its maximal preserving parameter interval; the exact irrational residual record reconstructs the Paper-I initial value.*
- (iv) *The Ω_0 positive ray is represented by $\mathbb{R}_{>0} \times \{\lambda\}$ under Θ , and its normalized direction is recognized exactly by any complete SRSD record.*
- (v) *Radial icosahedral moments, the five-design property, the degree-six obstruction, and the Euler–Maclaurin expansion are unchanged after exact SRSD transport and decoding.*
- (vi) *Projection onto any one component sort recovers the corresponding retained source-paper structure without strengthening its hypotheses.*

Proof. The SRSD inverse identities are Equation (1). Equation (18) follows by substituting the Paper-I definitions into the one-dimensional formula for E_h . Equation (19) is Euclidean division of Qd_k by S . These prove (i).

The exact finite-domain certificate gives Equation (20); invariance of D_{15} under both contractions gives equality of all iterates by induction. Subtracting the two definitions of the local residual gives Equation (21), proving (ii). Equations (9), (10), and (12) prove (iii).

The cone–simplex decomposition and the identities $(\phi, \pi) = \Omega_0(\lambda, \mu)$ give Equation (22). Injectivity of the frozen SRSD encoder gives Equation (23), proving (iv).

For (v), use the definition of pushforward measure and $D_s E_s = \text{id}$:

$$\int (Y \circ D_s) d(E_s)_\# \mu_f = \int Y \circ D_s \circ E_s d\mu_f = \int Y d\mu_f.$$

Taking Y successively to be a polynomial of degree at most five and then I_6 transports Equations (14)–(17) exactly.

Finally, $\mathfrak{G}$ was defined as a tuple of the component structures and the proved interface maps. Forgetting the other coordinates is a literal tuple projection. It neither changes a retained component’s operations nor adds a hypothesis, which proves (vi) and the claimed existence. $\square$

7 The finite reversible subobject

Let Fold_{15} map a finite byte string to its ordered 15-bit limbs, original byte length, and zero-padding count. Paper V proves that this map is injective onto the validated record set. For every limb $d_0 \in D_{15}$, iterate F_{q_0} to zero and retain the ordered Euclidean ledger

$$\text{Led}(d_0) = (0; \rho_0, \dots, \rho_{N-1}), \quad N \leq 10.$$

Define

$$\mathbf{W} = \left(\prod_{\text{limbs}} \text{Led} \right) \circ \text{Fold}_{15}.$$

Proposition 7.1 (Exact byte-to-witness injection). *The map $\mathbf{W}$ is injective on finite byte strings when limb order, original byte length, and padding count are retained. Its inverse first applies Equation (10) to every limb ledger and then applies the validated inverse of Fold_{15} .*

Proof. Each limb ledger is injective by exact Euclidean reconstruction. Their ordered product is therefore injective. The folding map is injective with its declared metadata, so the composition is injective. The described reverse operations are the two component inverses in reverse order. $\square$

This finite subobject must not be confused with the full real SRSD residual. The former uses bounded limbs and rational integer arithmetic; the latter is a two-sided exact decomposition of arbitrary real coordinates and does not imply a finite-bit representation.

8 The radial branch in resolved coordinates

Taking $Y = I_6$ in Equation (25) gives

$$\int_{\mathcal{R}_s} I_6(D_s e) d\hat{\mu}_{f,s}(e) = C_0 + C_2 f^{-2} + C_4 f^{-4} + O(f^{-6}). \quad (26)$$

The coefficients of the odd inverse powers through f^{-5} remain exactly zero. Likewise,

$$\int_{\mathcal{R}_s} I_6(D_s e) d\hat{\mu}_{\text{rad},s}(e) = C_0 > 0.$$

Corollary 8.1 (Coordinate invariance of the obstruction). *For every positive three-coordinate scale vector, the decoded resolved measures agree on all spherical harmonics through degree five and differ at degree six. Consequently the first distinguishing harmonic degree remains six after exact resolution.*

Proof. Equation (25) preserves every moment after decoding. Apply the five-design and positive degree-six moment statements of the radial branch. $\square$

This is a coordinate invariance statement. It does not derive the radial theorem from SRSD, and it does not turn the Bernstein certificate into a foundational SRSD theorem.

9 Dependency and status ledger

Source	Status in $\mathfrak{G}$	Exact role
Paper III	Sole frozen foundation	Defines E_s, D_s , exact resolution, fiber preservation, products, and the application boundary.
Paper I	Dynamical specialization	Supplies the real floor-quantized contraction, ordered residuals, reconstruction, and terminal dynamics.
Paper II	Floor-map specialization	Supplies rational Euclidean ledgers, orbit reconstruction, preserving parameter intervals, and finite-domain agreement.
Paper IV	Geometric-harmonic branch	Supplies the Class-I grids, radial limit, five-design property, degree-six obstruction, and asymptotic coefficients.
Paper V	Arithmetic and records	Supplies K_Ω , normal forms, the positive ray, canonical records, the 15-bit bridge, and cyclotomic/Fourier specializations.

The mathematical dependency arrows are therefore not a simple chain. Paper III governs exact coordinate resolution. Paper V supplies shared constants and exact finite interfaces. Papers I and II meet through the 15-bit quotient bridge. Paper IV is mathematically independent of those dynamics, although it shares ϕ , is expressible over the same real coordinate universe, and transports exactly through the frozen SRSD encoder.

10 Boundary statements

The global object makes the following distinctions explicit.

- (1) $q_0 \neq \lambda$. Their floor maps agree on the certified finite domain, not on all nonnegative integers.
- (2) Equality of quotient trajectories does not imply equality of residual ledgers; Equation (21) records the difference.
- (3) Exact SRSD resolution does not repair a noninjective representation, dynamical step, quotient, or record projection.
- (4) A canonical finite symbolic record for an element of K_Ω is not a finite positional expansion of its real value.
- (5) The radial harmonic theorem is not derived from Ω_0 , the rational floor dynamics, or SRSD. It is an independent branch carried without loss by the shared coordinate layer.
- (6) The number Ω_0 is specified exactly, but no physical privilege, universality, or necessity follows from the global assembly.
- (7) The multi-sorted packaging itself adds no novelty or priority claim to any component theorem.

11 Compact canonical form

The entire construction can be quoted in the following compact form:

$$\begin{aligned}
 \mathfrak{G} &= (\mathfrak{S}_{\text{SRSD}}, \mathfrak{A}_\Omega, \mathfrak{D}_{\text{quant}}, \mathfrak{D}_{\text{rat}}, \mathfrak{H}_{\text{ico}}, \mathfrak{C}_{\text{fin}}; \mathcal{J}), \\
 D_s E_s &= \text{id}, \\
 E_h(\lambda z_k) &= (z_{k+1}/h, \varepsilon_k), \\
 E_{S_0}(Q_0 d_k) &= (d_{k+1}, \rho_k), \\
 F_\lambda|_{D_{15}} &= F_{q_0}|_{D_{15}}, \quad \varepsilon_k^\lambda = \rho_k/S_0 + (\lambda - q_0)d_k, \\
 \Theta(c\phi, c\pi) &= (c, \lambda), \\
 \int Y \circ D_s d(E_s)_\# \nu &= \int Y d\nu, \\
 \mu_f(I_6) &= C_0 + C_2 f^{-2} + C_4 f^{-4} + O(f^{-6}), \quad C_0 > 0.
 \end{aligned}$$

Here ν may be μ_f or μ_{rad} , and the last line is the Paper-IV specialization of the preceding transport identity. This display is the defining mathematical core of the global object.

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