

OMEGA NAUGHT Typed Network

Exact Scalar Interfaces, Finite Lattice Conjugacy, Frozen SRSD
Encoding,
and the Independent Golden-Field Cubature Branch
Complete Proof Dossier with Embedded Exact Verifiers and Certificate

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Abstract

This dossier gives a typed and proof-complete audit of the exact interfaces among OMEGA Papers VII, V, I, II, and the frozen Paper-III SRSD representation, together with the independent $\mathbb{Q}(\sqrt{5})$ junction to the Class-I icosahedral cubature branch. It proves the corrected field-intersection theorem, mutual scalar reconstruction between Papers VII and V, equality of the Paper-V and Paper-I contraction constants, exact conjugacy of the Paper-I lattice step and Paper-II rational step on all 32768 unsigned 15-bit states, and exact factorization of the Paper-II quotient–remainder pair through the unchanged frozen SRSD encoder. The cubature branch remains logically independent. Its complete $f = 23, 24$ coefficient certificate, exact verifier, rank witnesses, positivity checks, and all 109 invariant moment identities are embedded in this same LaTeX source. The resulting theorems establish strictly positive H_3 -invariant rules through degrees 71 and 75; no maximality or all-frequency theorem is asserted.

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Part I

Typed network audit and interface proofs

1 Audit verdict and precise scope

The connected network is mathematically valid once every arrow is read at the level of the object on which it is actually defined. The rigorous form is

$$\boxed{\text{VII}_{\text{scal}} \xleftrightarrow[\Psi^{-1}]{\Psi} \text{V}_{\text{scal}} \xrightarrow{\lambda_{\Omega}=\lambda} \text{I} \xleftrightarrow[J_h]{\mathcal{D}_{15}} \text{II} \xrightarrow{E_S \circ M_Q} \text{III}_{\text{frozen}}.} \quad (\text{N1})$$

Here the middle double arrow denotes conjugacy of restricted step maps, not equivalence of the complete contents of Papers I and II. The independent cubature junction is

$$\boxed{K_{\text{coord}} \hookleftarrow K_5 \hookrightarrow K_{\Omega}, \quad K_5 \longrightarrow \text{IV/VI} \longrightarrow (f = 23, 24) \text{ exact certificates.}} \quad (\text{N2})$$

The first two arrows in (N2) are field inclusions; the final arrow is proof dependence inside the cubature branch. There is no causal arrow from Omega arithmetic, the floor map, or SRSD into the cubature existence proof.

Interface	Exact mathematical type	Claim boundary
VII–V	mutually inverse reconstruction of distinguished scalar packages	does not reconstruct a geometric realization without center, scale, frame, and construction data
V–I	equality of the contraction scalar	parameter identification, not theorem dependence
I–II	conjugacy of restricted one-step maps on $\mathcal{D}_{15}$	not equality outside the certified finite domain
II–III	exact quotient–residual factorization of $d \mapsto Qd$	an external use of the frozen encoder, not a new SRSD primitive
VII/V–IV/VI	common coefficient subfield $K_5 = \mathbb{Q}(\sqrt{5})$	representation compatibility, not a cubature proof premise
IV/VI– $f = 23, 24$	invariant reduction plus exact finite certificates	existence through degrees 71 and 75, with no maximality claim

Theorem 1 (Typed network theorem). *With the definitions below, all of the following statements hold.*

1. *The coordinate-coefficient field of Paper VII and the Omega arithmetic field meet exactly in the golden field:*

$$K_{\text{coord}} \cap K_{\Omega} = K_5 = \mathbb{Q}(\sqrt{5}).$$

The intrinsic full constant field of Paper VII instead contains K_{Ω} and satisfies

$$K_{\text{VII}}^{\text{const}} = K_{\Omega}(s, c), \quad K_{\text{VII}}^{\text{const}} \cap K_{\Omega} = K_{\Omega}.$$

2. *The distinguished scalar packages (ϕ, ε) and (τ_{Ω}, λ) reconstruct one another exactly.*

3. Once independent realization data (α, a, u, v) are supplied, the Paper-VII marked coordinates are reconstructed exactly from (τ_Ω, λ) .
4. For every $h > 0$, the Paper-I quantized step restricted to the lattice $h\mathcal{D}_{15}$ is conjugate to the Paper-II rational integer step by $J_h(d) = hd$.
5. The Paper-II quotient and Euclidean remainder are exactly the frozen scalar SRSD encoding of Qd at scale S .
6. The cubature coefficient field K_5 embeds naturally into K_Ω , but the certified $f = 23, 24$ cubature theorems remain logically independent of Omega arithmetic, the floor dynamics, and SRSD.

The remaining sections prove each clause and record the finite certificates.

2 Notation and the two necessary corrections

Set

$$\phi = \frac{1 + \sqrt{5}}{2}, \quad \varepsilon = \frac{\pi}{648000}, \quad \delta = \frac{\varepsilon}{2} = \frac{\pi}{1296000},$$

and

$$s = \sin \delta, \quad c = \cos \delta.$$

For the Omega arithmetic package put

$$\Omega_0 = \phi + \pi, \quad \tau_\Omega = \Omega_0^{-1}, \quad \lambda = \frac{\phi}{\Omega_0}, \quad \mu = \frac{\pi}{\Omega_0} = 1 - \lambda.$$

2.1 Correction 1: intrinsic constants versus arbitrary realization data

The relevant fields are

$$\begin{aligned} K_5 &= \mathbb{Q}(\sqrt{5}), & K_\Omega &= \mathbb{Q}(\pi, \sqrt{5}), \\ K_{\text{coord}} &= \mathbb{Q}(\sqrt{5}, s, c), & K_{\text{VII}}^{\text{const}} &= \mathbb{Q}(\pi, \sqrt{5}, s, c). \end{aligned}$$

The term *coordinate-coefficient field* means the coefficients of the marked-point formula relative to the formal realization data (α, a, u, v) . It does not include arbitrary Cartesian coordinates of α, u, v or an arbitrary real scale a . Those choices can enlarge the field of a particular realization and therefore must not be inserted into the intersection theorem.

2.2 Correction 2: the two published uses of τ

The quantity $\tau_\Omega = 1/\Omega_0$ is a distinguished Omega constant. In Paper I, however, the notation $\tau > 0$ denotes an arbitrary quantization scale. Throughout this interface theorem only, Paper I's published scale is renamed

$$\boxed{h > 0}, \quad Q_h(y) = h \left\lfloor \frac{y}{h} \right\rfloor.$$

This is a notational relabeling, not a modification of Paper I. The geometric dummy parameter on a Paper-VII baseline segment is likewise not the contraction scalar λ ; when needed it may be called t .

3 The exact field junction

Lemma 1 (Cyclotomic algebraicity of the half-angle data). *The numbers $s = \sin \delta$ and $c = \cos \delta$ are algebraic over $\mathbb{Q}$.*

Proof. Let

$$N = 2592000, \quad \zeta = e^{2\pi i/N}.$$

Since $\delta = \pi/1296000 = 2\pi/N$, one has $e^{i\delta} = \zeta$ and hence

$$c = \frac{\zeta + \zeta^{-1}}{2}.$$

Moreover $4 \mid N$ and therefore

$$i = \zeta^{N/4} = \zeta^{648000}.$$

It follows that

$$s = \frac{\zeta - \zeta^{-1}}{2i}.$$

Thus $s, c \in \mathbb{Q}(\zeta)$, a cyclotomic number field, so both are algebraic over $\mathbb{Q}$. $\square$

Lemma 2 (Constant-field fact). *Let k be a field and let t be transcendental over k . Every element of $k(t)$ that is algebraic over k belongs to k .*

Proof. Write $x = f(t)/g(t)$ with coprime $f, g \in k[T]$ and $g \neq 0$. If x is algebraic over k , then $k(x)/k$ is algebraic. The equation

$$f(T) - xg(T) = 0$$

shows that t is algebraic over $k(x)$ unless this polynomial is identically zero. Algebraicity would then be transitive and make t algebraic over k , a contradiction. Hence $f(T) - xg(T)$ is the zero polynomial. Comparing any nonzero coefficient of g shows $x \in k$. $\square$

Theorem 2 (Corrected field-intersection theorem). *One has*

$$\boxed{K_{\text{coord}} \cap K_{\Omega} = K_5.}$$

In contrast,

$$\boxed{K_{\text{VII}}^{\text{const}} = K_{\Omega}(s, c), \quad K_{\text{VII}}^{\text{const}} \cap K_{\Omega} = K_{\Omega}.}$$

Proof. The preceding cyclotomic lemma shows that K_{coord}/K_5 is algebraic. The number π is transcendental over K_5 : otherwise, because $K_5/\mathbb{Q}$ is algebraic, transitivity would make π algebraic over $\mathbb{Q}$, contrary to Lindemann's theorem. Consequently

$$K_{\Omega} = K_5(\pi)$$

is a rational-function field in the transcendental element π over K_5 .

If $x \in K_{\text{coord}} \cap K_{\Omega}$, then x is algebraic over K_5 and also belongs to $K_5(\pi)$. The constant-field lemma gives $x \in K_5$. The reverse inclusion $K_5 \subseteq K_{\text{coord}} \cap K_{\Omega}$ is immediate. This proves the first equality.

The second line follows directly from the definitions:

$$K_{\text{VII}}^{\text{const}} = \mathbb{Q}(\pi, \sqrt{5}, s, c) = K_{\Omega}(s, c),$$

so $K_{\Omega} \subseteq K_{\text{VII}}^{\text{const}}$ and the stated intersection is K_{Ω} . $\square$

Remark 1. The first boxed equality is the nontrivial theorem. The second prevents the incorrect extension of that theorem from Paper VII's algebraic coordinate coefficients to all of its intrinsic constants, since $\pi = 648000\varepsilon$ is already present in Paper VII.

4 Paper VII and Paper V: exact scalar reconstruction

Theorem 3 (Mutually inverse scalar interface). *Define*

$$\Psi(\phi, \varepsilon) = \left(\frac{1}{\phi + 648000\varepsilon}, \frac{\phi}{\phi + 648000\varepsilon} \right) = (\tau_\Omega, \lambda).$$

On the image of this map, the inverse is

$$\Psi^{-1}(\tau_\Omega, \lambda) = \left(\frac{\lambda}{\tau_\Omega}, \frac{1 - \lambda}{648000\tau_\Omega} \right) = (\phi, \varepsilon).$$

Thus the two distinguished scalar packages reconstruct one another exactly.

Proof. Paper VII gives $\pi = 648000\varepsilon$, so

$$\Omega_0 = \phi + 648000\varepsilon, \quad \tau_\Omega = \frac{1}{\phi + 648000\varepsilon}, \quad \lambda = \frac{\phi}{\phi + 648000\varepsilon}.$$

It follows immediately that

$$\frac{\lambda}{\tau_\Omega} = \phi, \quad \frac{1 - \lambda}{\tau_\Omega} = \pi, \quad \frac{1 - \lambda}{648000\tau_\Omega} = \varepsilon.$$

Conversely, start with (τ_Ω, λ) in the displayed image and apply the proposed inverse. The reconstructed denominator is

$$\frac{\lambda}{\tau_\Omega} + 648000 \frac{1 - \lambda}{648000\tau_\Omega} = \frac{1}{\tau_\Omega}.$$

Applying Ψ therefore returns τ_Ω and λ . Both compositions are the identity on the declared scalar packages. $\square$

Corollary 1 (Reconstruction of all Omega scalars). *The interface also gives*

$$\begin{aligned} \Omega_0 &= \frac{1}{\tau_\Omega}, & \mu &= 1 - \lambda, \\ \phi &= \frac{\lambda}{\tau_\Omega}, & \pi &= \frac{1 - \lambda}{\tau_\Omega}, & \sqrt{5} &= 2 \frac{\lambda}{\tau_\Omega} - 1. \end{aligned}$$

Consequently

$$K_\Omega = \mathbb{Q}(\tau_\Omega, \lambda) = \mathbb{Q}(\pi, \sqrt{5}).$$

Proof. Only the last field equality requires comment. The displayed reconstruction formulas give $\mathbb{Q}(\pi, \sqrt{5}) \subseteq \mathbb{Q}(\tau_\Omega, \lambda)$. Conversely, τ_Ω and λ are rational expressions in π and $\sqrt{5}$, giving the reverse inclusion. $\square$

4.1 Reconstruction of the marked Paper-VII coordinates

Let α be a center, let $a > 0$, and let (u, v) be an ordered orthonormal frame in an oriented Euclidean plane. From (τ_Ω, λ) define

$$\delta = \frac{1 - \lambda}{1296000\tau_\Omega}, \quad s = \sin\left(\frac{1 - \lambda}{1296000\tau_\Omega}\right), \quad c = \cos\left(\frac{1 - \lambda}{1296000\tau_\Omega}\right).$$

Then

$$x_0 = s, \quad x_1 = 1 + s, \quad x_2 = \phi^2 + s = 1 + \frac{\lambda}{\tau_\Omega} + s,$$

and the six ordered marks are

$$\boxed{p_{j,\sigma} = \alpha + a(cu + \sigma x_j v), \quad j \in \{0, 1, 2\}, \quad \sigma \in \{-1, +1\}.} \quad (\text{N3})$$

This is an exact realization interface. It proves that Paper-V scalar data, together with the independent realization data and Paper-VII construction rules, determines the Paper-VII marked coordinates. It does not say that Paper V alone proves the Paper-VII geometric theorems.

5 Paper V and Paper I: the identical contraction scalar

Paper I defines

$$\lambda_\Omega = \frac{\phi}{\phi + \pi}.$$

The Omega arithmetic package defines

$$\lambda = \frac{\phi}{\Omega_0} = \frac{\phi}{\phi + \pi}.$$

Therefore

$$\boxed{\lambda_\Omega = \lambda.} \quad (\text{N4})$$

This is equality of real numbers, not an approximation and not an import of one paper's theorem package into another.

With Paper I's arbitrary quantizer scale renamed $h > 0$, define

$$Q_h(y) = h \left\lfloor \frac{y}{h} \right\rfloor, \quad T_h(z) = Q_h(\lambda z).$$

For every $d \in \mathbb{Z}$,

$$T_h(hd) = h \left\lfloor \frac{\lambda hd}{h} \right\rfloor = h \lfloor \lambda d \rfloor,$$

and hence

$$\boxed{h^{-1}T_h(hd) = \lfloor \lambda d \rfloor.} \quad (\text{N5})$$

6 Paper I and Paper II: exact finite-domain conjugacy

Set

$$S = 10^{15}, \quad Q = 339949771344778, \quad q = \frac{Q}{S},$$

$$\mathcal{D}_{15} = \{0, 1, \dots, 32767\}, \quad F_q(d) = \left\lfloor \frac{Qd}{S} \right\rfloor, \quad F_\lambda(d) = \lfloor \lambda d \rfloor.$$

6.1 Exact enclosure of λ

The following two displayed decimals denote exact rational numbers with denominator 10^{60} :

$$L = \frac{339949771344778929576702809321279193551009905014259416689444}{10^{60}},$$

$$U = \frac{339949771344778929576702809321279193551009905014259416689445}{10^{60}}.$$

Lemma 3 (Exact enclosure). *One has*

$$q < L < \lambda < U < q + 10^{-15}.$$

In particular $Q = \lfloor S\lambda \rfloor$.

Proof. For even N , put

$$A_N(n) = \sum_{k=0}^{N-1} \frac{(-1)^k}{(2k+1)n^{2k+1}}.$$

The alternating-series theorem gives

$$A_{60}(5) < \arctan(1/5) < A_{60}(5) + \frac{1}{121 \cdot 5^{121}},$$

$$A_{14}(239) < \arctan(1/239) < A_{14}(239) + \frac{1}{29 \cdot 239^{29}}.$$

Use Machin's identity

$$\pi = 16 \arctan(1/5) - 4 \arctan(1/239).$$

Substituting the lower bound in the positive term and the upper bound in the subtracted term gives an exact rational $\pi_- < \pi$; the opposite choices give $\pi < \pi_+$.

For $\sqrt{5}$, let

$$A = 2236067977499789696409173668731276235440618359,$$

$$B = 611525724270897245410520925637804899414414408,$$

$$R = 10^{45}A + B.$$

Direct integer squaring gives

$$R^2 < 5 \cdot 10^{180} < (R+1)^2,$$

so

$$\frac{R}{10^{90}} < \sqrt{5} < \frac{R+1}{10^{90}}.$$

Let $\phi_- < \phi < \phi_+$ be the induced rational bounds. The function $x/(x+y)$ is increasing in $x > 0$ and decreasing in $y > 0$, whence

$$\frac{\phi_-}{\phi_- + \pi_+} < \lambda < \frac{\phi_+}{\phi_+ + \pi_-}.$$

Exact rational cross-multiplication, carried out by the embedded verifier, proves that the left member is greater than L and the right member is less than U . The same exact arithmetic verifies $SL > Q$ and $SU < Q + 1$. Therefore

$$q = Q/S < L < \lambda < U < (Q+1)/S = q + 10^{-15}.$$

No floating-point value is used in these comparisons. □

Theorem 4 (Certified floor equality). *For every $d \in \mathcal{D}_{15}$,*

$$\boxed{F_\lambda(d) = F_q(d)}. \quad (\text{N6})$$

Proof. For each of the 32768 integers in the domain, let

$$m_d = \left\lfloor \frac{Qd}{S} \right\rfloor.$$

The embedded exact-rational verifier checks

$$m_d \leq Ld \leq Ud < m_d + 1$$

for every such d . Lemma 3 then gives

$$m_d \leq \lambda d < m_d + 1,$$

so $\lfloor \lambda d \rfloor = m_d = \lfloor Qd/S \rfloor$.

There is also a compact independent boundary-margin certificate. Since $q < \lambda < U$, only an upward integer boundary can be crossed. The smallest distance from qd to its next integer over the complete domain is

$$\Delta_{\min} = \frac{34535026732}{10^{15}}, \quad \text{attained at } d = 20706.$$

Furthermore

$$U - q = \frac{929576702809321279193551009905014259416689445}{10^{60}},$$

and exact integer arithmetic gives

$$\begin{aligned} 32767(10^{60}(U - q)) &= 30459439820953030355335085941557602238306663044315 \\ &< 34535026732 \cdot 10^{45}. \end{aligned}$$

After division by 10^{60} ,

$$32767(U - q) < \Delta_{\min}.$$

Thus for every $d \in \mathcal{D}_{15}$ the displacement from qd to λd is smaller than the available upward gap, and no floor boundary can be crossed. The minimum-gap calculation is itself checked by exhaustive integer residues in the embedded verifier. $\square$

Theorem 5 (Restricted conjugacy). *For every $h > 0$, let*

$$J_h : \mathcal{D}_{15} \longrightarrow h\mathcal{D}_{15}, \quad J_h(d) = hd.$$

Then

$$\boxed{T_h \circ J_h = J_h \circ F_q \quad \text{on } \mathcal{D}_{15}.} \quad (\text{N7})$$

Equivalently,

$$\boxed{T_h|_{h\mathcal{D}_{15}} = J_h \circ F_q \circ J_h^{-1}.} \quad (\text{N8})$$

The same identity holds for every iterate.

Proof. Equations (N5) and (N6) give

$$T_h(J_h(d)) = h\lfloor \lambda d \rfloor = hF_q(d) = J_h(F_q(d)).$$

Because $0 \leq F_q(d) \leq d \leq 32767$, the set $\mathcal{D}_{15}$ is forward invariant. The map J_h is a bijection from $\mathcal{D}_{15}$ onto $h\mathcal{D}_{15}$, so the conjugacy formula follows. Induction gives

$$T_h^n \circ J_h = J_h \circ F_q^n \quad (n \geq 0).$$

$\square$

7 Paper II and frozen Paper III: exact quotient–residual encoding

For the scalar frozen SRSD encoder and decoder at scale $S > 0$, write

$$E_S(y) = \left(\left\lfloor \frac{y}{S} \right\rfloor, y - S \left\lfloor \frac{y}{S} \right\rfloor \right),$$

$$D_S(d', \rho) = Sd' + \rho, \quad d' \in \mathbb{Z}, \quad 0 \leq \rho < S.$$

Define the Paper-II Euclidean remainder

$$\rho_q(d) = Qd - SF_q(d)$$

and the external multiplication map $M_Q(d) = Qd$.

Theorem 6 (Exact SRSD factorization). *For every nonnegative integer d ,*

$$\boxed{(E_S \circ M_Q)(d) = E_S(Qd) = (F_q(d), \rho_q(d))}. \quad (\text{N9})$$

Moreover

$$\boxed{D_S(F_q(d), \rho_q(d)) = Qd}, \quad (\text{N10})$$

and therefore

$$\boxed{d = \frac{D_S(F_q(d), \rho_q(d))}{Q}}. \quad (\text{N11})$$

Proof. Substitution into the frozen encoder gives

$$\begin{aligned} E_S(Qd) &= \left(\left\lfloor \frac{Qd}{S} \right\rfloor, Qd - S \left\lfloor \frac{Qd}{S} \right\rfloor \right) \\ &= (F_q(d), \rho_q(d)). \end{aligned}$$

Applying the frozen decoder yields

$$D_S(F_q(d), \rho_q(d)) = SF_q(d) + \rho_q(d) = Qd.$$

Since $Q > 0$, division by Q gives (N11). □

Corollary 2 (Exact ordered ledger reconstruction). *If*

$$d_{k+1} = F_q(d_k), \quad \rho_k = Qd_k - Sd_{k+1},$$

then

$$Qd_k = Sd_{k+1} + \rho_k, \quad 0 \leq \rho_k < S,$$

and for $N \geq 1$,

$$\boxed{Q^N d_0 = S^N d_N + \sum_{k=0}^{N-1} \rho_k S^k Q^{N-1-k}}. \quad (\text{N12})$$

Thus d_N and the ordered remainder ledger $(\rho_0, \dots, \rho_{N-1})$ reconstruct d_0 exactly.

Proof. The range of ρ_k is the Euclidean-remainder bound. The one-step inverse is

$$d_k = \frac{Sd_{k+1} + \rho_k}{Q}.$$

For the closed formula, multiply the $k = 0$ identity by Q^{N-1} and successively substitute $Qd_1 = Sd_2 + \rho_1$, then $Qd_2 = Sd_3 + \rho_2$, and so on. The displayed sum results. Reversing these substitutions reconstructs the predecessors uniquely. $\square$

Remark 2 (Frozen boundary). The operation in (N9) is a specialization of the already defined frozen SRSD encoder, applied externally to the scalar Qd at scale S . Multiplication by Q and the final division by Q are external maps. No SRSD definition, ontology, or theorem is changed.

8 The golden-field cubature junction

The Paper-V reconstruction identity gives

$$\sqrt{5} = 2\frac{\lambda}{\tau_\Omega} - 1.$$

Define

$$\iota : K_5 \longrightarrow K_\Omega, \quad \iota(a + b\sqrt{5}) = a + b\left(2\frac{\lambda}{\tau_\Omega} - 1\right).$$

Proposition 1 (Natural coefficient-field inclusion). *The map ι is the natural injective field homomorphism $\mathbb{Q}(\sqrt{5}) \hookrightarrow \mathbb{Q}(\pi, \sqrt{5})$.*

Proof. Because

$$2\frac{\lambda}{\tau_\Omega} - 1 = 2\phi - 1 = \sqrt{5},$$

the formula is simply $\iota(x) = x$ under the usual realization of both fields inside $\mathbb{R}$. It preserves addition, multiplication, and 1, and its kernel is zero. $\square$

The Class-I cubature matrices, target moments, and certified weights lie in K_5 . They may therefore be represented inside K_Ω through ι . This extension of scalars preserves exact equalities. It is not used to prove the equalities: the cubature verifier works directly in K_5 and makes no use of π , Ω_0 , λ , the finite floor map, or SRSD.

The strongest field diagram is consequently

$$\boxed{K_{\text{coord}} \hookleftarrow K_5 \hookrightarrow K_\Omega, \quad \text{cubature coefficients} \subset K_5.} \quad (\text{N13})$$

Together with Theorem 2, this states both the exact junction and its limit.

9 Finite cubature audit data

The complete mathematical proof, all 117 exact orbit masses, the exact arithmetic verifier, the 109-row residual ledger, and the modular rank witnesses occur in Part II of this same LaTeX source. The elementary sizes are recorded here because they are also independently checkable.

For

$$\Lambda_f = \{(a, b, c) \in \mathbb{Z}_{\geq 0}^3 : a \geq b \geq c, a + b + c = f\},$$

one has

$$|\Lambda_f| = \sum_{c=0}^{\lfloor f/3 \rfloor} \left(\left\lfloor \frac{f-c}{2} \right\rfloor - c + 1 \right).$$

Therefore

$$\begin{aligned} |\Lambda_{23}| &= 12 + 11 + 9 + 8 + 6 + 5 + 3 + 2 = 56, \\ |\Lambda_{24}| &= 13 + 11 + 10 + 8 + 7 + 5 + 4 + 2 + 1 = 61. \end{aligned}$$

The complete radial Class-I grids satisfy

$$|X_f| = 10f^2 + 2,$$

so

$$|X_{23}| = 5292, \quad |X_{24}| = 5762.$$

The invariant rows are pairs $(r, s) \in \mathbb{Z}_{\geq 0}^2$ satisfying $6r + 10s \leq t$. For $t = 71$, this is $3r + 5s \leq 35$, and

$$12 + 11 + 9 + 7 + 6 + 4 + 2 + 1 = 52.$$

For $t = 75$, it is $3r + 5s \leq 37$, and

$$13 + 11 + 10 + 8 + 6 + 5 + 3 + 1 = 57.$$

The modular rank certificate uses

$$p = 1000000000000000031,$$

and

$$\rho_+ = 968101896520624317, \quad \rho_- = 31898103479375714.$$

Exact modular arithmetic gives

$$\rho_+ + \rho_- = p, \quad \rho_+^2 \equiv \rho_-^2 \equiv 5 \pmod{p}.$$

After deleting the four declared free columns, the determinant residues are

	$\sqrt{5} \mapsto \rho_+$	$\sqrt{5} \mapsto \rho_-$
$f = 23$	650097745717062046	139778200766197651
$f = 24$	550949013298233594	966277299582556251

and all four are nonzero. Thus the selected exact square minors cannot have zero determinant. Their ranks are therefore 52 and 57, the full row ranks permitted by their numbers of rows.

A fresh replay on 5 September 2026 returned

```
PASS f=23 tau=71 nodes=5292 orbits=56 rows=52 rank=52 free=4 min=(21, 1, 1) ~
→ 0.003777725693780652542084082262449790124026578124021415259901
PASS f=24 tau=75 nodes=5762 orbits=61 rows=57 rank=57 free=4 min=(23, 1, 0) ~
→ 0.003412253320984930573099634686301743266126075350598455734277
ALL PASS: 109 exact invariant moment identities; strict positivity and rank certified over
→ Q(sqrt(5)).
certificate_sha256 e886510ec03afe014ad143d6d0b45677fc2bb58a7df495374b5780cf79d5b5af
```

This is a same-environment exact replay and not an external clean-room reproduction or peer review.

The resulting certified claims are exactly

X_{23} admits a strictly positive H_3 -invariant cubature rule through degree 71,

and

X_{24} admits a strictly positive H_3 -invariant cubature rule through degree 75.

Neither maximality of those degrees nor a uniform all-frequency theorem is asserted.

10 Completion of the network proof

Proof of Theorem 1. Clause 1 is Theorem 2. Clause 2 is Theorem 3 and its corollary. Clause 3 is the marked coordinate formula (N3). Clause 4 is Theorem 5, whose finite arithmetic premise is Theorem 4. Clause 5 is Theorem 6. Clause 6 is the natural field inclusion (N13) together with the self-contained cubature proof in Part II. Every arrow in (N1)–(N2) has therefore been supplied with a domain, codomain, map, and proved identity. $\square$

The final defensible summary is

$$\begin{array}{l}
 \text{VII} \leftrightarrow \text{V} : \text{exact scalar reconstruction,} \\
 \text{V} \rightarrow \text{I} : \text{identical contraction scalar,} \\
 \text{I} \leftrightarrow \text{II} : \text{restricted lattice conjugacy on } \mathcal{D}_{15}, \\
 \text{II} \rightarrow \text{III} : \text{exact external quotient–residual encoding,} \\
 \text{VII/V and IV/VI} : \text{shared golden coefficient field, without proof dependence.}
 \end{array} \tag{N14}$$

This is stronger than a statement that the papers are merely related. It is a typed diagram of exact interfaces between distinct theorem packages, not a claim that the packages form one retrospective theory.

11 Embedded network verifier and source audit

The opening block `verify_OMEGA_typed_network_v1.0.py` is part of this LaTeX source. It reconstructs the Machin and integer-square-root bounds using exact rational and integer arithmetic, checks all 32768 interval inequalities, checks the independent minimum-margin certificate, recomputes the grid/orbit/row counts, tests primality of the modular witness modulus, checks both square roots of 5, and checks that all four determinant residues are nonzero.

The exact sources used for this audit were pinned by the following SHA-256 digests:

Source	SHA-256
OMEGA_Paper_VII_v1.6.0.tex	155ea07227f5e498004fc7fe36113283763450cfa cd85fef39518a9502719768
Newman_Quantized_Omega_Contraction_Theorem_Zenodo_21830285.tex	acd4d8bf2277ad5dd5466c06ed6da077b01595231 4eab9dd2cb757a0e5a0693c
Final2SUBOMEGA_NAUGHT_Reference_Note_v1.0.tex	16c52558f700964dbd3f71c25ce0e2e353c9ed2ea c4278a74d64062713aa51eb
Paper_IV_Draft_7.tex (Omega-field theorem source in the supplied corpus)	45623c81f47305bea15582ccfa70691f490751053 fbaf92a56234404a9e7f332
ClassI_Cubature_f23_f24_exact_certificate_v1.1.json	e886510ec03afe014ad143d6d0b45677fc2bb58a7 df495374b5780cf79d5b5af
verify_ClassI_f23_f24_exact_v1.1.py	41d83727473850ebf0b0b00f57f38a64c08d984 9170a880b23163769509ce1
ClassI_Cubature_f23_f24_verification_ledger_v1.1.txt	4d56dc2c65466fc89e6c275fdc38012dbca2c5b74 456cf045e7ceedb30a38e6e

Paper VII is cited here at version 1.6.0, not the superseded v1.5 source. The field reconstruction and 15-bit bridge are also restated and proved in the public Reference Note v1.0. The complete cubature

certificate and verifier are embedded below, so the two finite existence claims do not depend on an unprinted list of decimal weights.

Part II

Exact $f = 23, 24$ cubature certificate and proof

12 Exact statement and scope

Theorem 7 (Frequency 23). *Let $X_{23} \subset \mathbb{S}^2$ be the complete radial Class-I icosahedral grid. There exist total orbit masses $W_\lambda \in \mathbb{Q}(\sqrt{5})$, one for each full icosahedral reflection-group orbit, such that*

$$W_\lambda > 0, \quad \sum_{\lambda} W_\lambda = 1,$$

and the associated orbit-constant rule integrates every spherical polynomial of degree at most 71 exactly.

Theorem 8 (Frequency 24). *Let $X_{24} \subset \mathbb{S}^2$ be the complete radial Class-I icosahedral grid. There exist total orbit masses $W_\lambda \in \mathbb{Q}(\sqrt{5})$, one for each full icosahedral reflection-group orbit, such that*

$$W_\lambda > 0, \quad \sum_{\lambda} W_\lambda = 1,$$

and the associated orbit-constant rule integrates every spherical polynomial of degree at most 75 exactly.

These are existence and exactness-through-degree statements on two fixed node sets. They do not say that 71 or 75 is maximal, and they do not prove a uniform assertion for all frequencies. The continuation from $f \geq 25$ remains open. No property of Ω_0 , the rational floor map, or SRSD is used in this proof; those are separate branches.

13 The fixed Class-I grids

Put

$$\varphi = \frac{1 + \sqrt{5}}{2}, \quad R^2 = \varphi + 2,$$

and use the twelve standard icosahedron vertices

$$V = \{(0, \pm 1, \pm \varphi), (\pm 1, \pm \varphi, 0), (\pm \varphi, 0, \pm 1)\}.$$

On one face choose

$$v_1 = (0, 1, \varphi), \quad v_2 = (-1, \varphi, 0), \quad v_3 = (1, \varphi, 0).$$

For

$$\Lambda_f = \{(a, b, c) : a \geq b \geq c \geq 0, a + b + c = f\}$$

define

$$p_{a,b,c} = av_1 + bv_2 + cv_3, \quad x_{a,b,c} = \frac{p_{a,b,c}}{\|p_{a,b,c}\|}.$$

Let H_3 be the full icosahedral reflection group and $\mathcal{O}_{a,b,c} = H_3 x_{a,b,c}$. Then

$$X_f = \bigcup_{(a,b,c) \in \Lambda_f} \mathcal{O}_{a,b,c}.$$

The inequalities defining Λ_f choose exactly one representative from each orbit.

Lemma 4 (Grid cardinality). *For every positive integer f ,*

$$|X_f| = 10f^2 + 2.$$

Proof. The radial projection is injective on the subdivided boundary of a convex icosahedron. Count the 12 original vertices, the $f - 1$ new points on each of 30 edges, and the $(f - 1)(f - 2)/2$ interior lattice points on each of 20 faces:

$$12 + 30(f - 1) + 20 \frac{(f - 1)(f - 2)}{2} = 10f^2 + 2.$$

Thus $|X_{23}| = 5292$ and $|X_{24}| = 5762$. □

For total orbit masses $W_{a,b,c}$ define

$$Q_f[g] = \sum_{(a,b,c) \in \Lambda_f} \frac{W_{a,b,c}}{|\mathcal{O}_{a,b,c}|} \sum_{x \in \mathcal{O}_{a,b,c}} g(x).$$

The orbit sizes are 12 at a vertex; 30 at an edge midpoint; 60 at a nonmidpoint edge point; and for a face-interior point they are 20, 60, or 120 according as all three barycentric entries are equal, exactly two are equal, or all are distinct. Consequently positive orbit masses give positive per-node weights.

Throughout, σ is normalized spherical area, so $\int_{\mathbb{S}^2} 1 \, d\sigma = 1$.

14 Invariant moment system

For $n = 6, 10$ define the power-sum invariants used by the certificate by

$$U_n(x) = \sum_{v \in V} \frac{(x \cdot v)^n}{(R^2)^{n/2}}.$$

These are invariant under H_3 . For a target degree t , put

$$\mathcal{I}_t = \{(r, s) \in \mathbb{Z}_{\geq 0}^2 : 6r + 10s \leq t\}.$$

Writing $\lambda = (a, b, c) \in \Lambda_f$, define

$$A_{(r,s),\lambda} = U_6(x_\lambda)^r U_{10}(x_\lambda)^s, \quad b_{r,s} = \int_{\mathbb{S}^2} U_6^r U_{10}^s \, d\sigma.$$

The certificate equation is

$$AW = b.$$

Lemma 5 (Invariant reduction). *An H_3 -invariant rule Q is exact on every spherical polynomial of degree at most t if and only if it is exact on $U_6^r U_{10}^s$ for every $(r, s) \in \mathcal{I}_t$.*

Proof. For any polynomial p , average over the group:

$$(\mathcal{R}p)(x) = \frac{1}{|H_3|} \sum_{g \in H_3} p(gx).$$

Because both Q and spherical area are H_3 -invariant, $Q[p] = Q[\mathcal{R}p]$ and $\int p \, d\sigma = \int \mathcal{R}p \, d\sigma$. Thus only invariant polynomials need be tested. The invariant ring of the reflection group H_3 has homogeneous basic degrees 2, 6, 10. On the unit sphere the degree-2 generator is constant. The displayed power sums U_6, U_{10} are related to degree-6 and degree-10 basic invariants by an invertible triangular change using lower-degree invariants. Hence their monomials with $6r + 10s \leq t$ span the restricted invariant polynomials through degree t . This is the invariant-cubature reduction. See [9]. □

Lemma 6 (Exact spherical monomial moments). *For normalized spherical area,*

$$\int_{\mathbb{S}^2} x^{2i} y^{2j} z^{2k} d\sigma = \frac{(2i-1)!!(2j-1)!!(2k-1)!!}{(2i+2j+2k+1)!!},$$

with $(-1)!! = 1$; any monomial with an odd exponent has mean zero.

Proof. Odd monomials vanish by coordinate reflection. For even exponents, compare the Cartesian factorization of the Gaussian integral $\int_{\mathbb{R}^3} x^{2i} y^{2j} z^{2k} e^{-\|x\|^2} dx dy dz$ with its polar-coordinate factorization. Cancelling the common radial gamma factor gives the stated double-factorial ratio; normalization by the sphere's area makes the constant moment equal to 1. $\square$

Lemma 7 (Quadratic-field closure). *Every entry of A and b lies in $\mathbb{Q}(\sqrt{5})$.*

Proof. Each unprojected $p_{a,b,c}$ has coordinates in $\mathbb{Q}(\sqrt{5})^3$, and $\|p_{a,b,c}\|^2 \in \mathbb{Q}(\sqrt{5})$. Because the invariant degrees are even, radial normalization introduces only integer powers of this squared norm and of $R^2 \in \mathbb{Q}(\sqrt{5})$. Thus $U_6(x_\lambda), U_{10}(x_\lambda) \in \mathbb{Q}(\sqrt{5})$. Expanding $U_6^r U_{10}^s$ and applying the preceding exact monomial formula keeps every spherical moment in $\mathbb{Q}(\sqrt{5})$. $\square$

15 Exact sign decisions in $\mathbb{Q}(\sqrt{5})$

Write $z = a + b\sqrt{5}$ with $a, b \in \mathbb{Q}$. If a, b have the same weak sign, the sign is immediate. If their signs differ, compare a^2 with $5b^2$; for example, when $a > 0 > b$,

$$a + b\sqrt{5} > 0 \iff a^2 > 5b^2.$$

The other mixed-sign case is analogous. Since 5 is not a rational square, $a^2 = 5b^2$ with rational a, b forces $a = b = 0$. Therefore every sign and pairwise comparison used below is decided with rational integer arithmetic, without approximating $\sqrt{5}$.

16 System sizes and full-rank witnesses

The target convention is

$$\tau_f = \begin{cases} 3f + 2, & f \text{ odd,} \\ 3f + 3, & f \text{ even.} \end{cases}$$

Direct enumeration gives

$$|\Lambda_{23}| = 56, \quad |\mathcal{I}_{71}| = 52, \quad |\Lambda_{24}| = 61, \quad |\mathcal{I}_{75}| = 57.$$

Indeed $|\mathcal{I}_{71}| = \#\{(r, s) : 3r + 5s \leq 35\} = 52$ and $|\mathcal{I}_{75}| = \#\{(r, s) : 3r + 5s \leq 37\} = 57$.

f	$ X_f $	orbits	rows	rank	affine dimension
23	5292	56	52	52	4
24	5762	61	57	57	4

The four declared free coordinates at each frequency are:

f	orbit	exact orbit mass
23	(17, 5, 1)	82833/5496019
23	(16, 6, 1)	776496/85219693
23	(13, 9, 1)	67140/3095507
23	(10, 8, 5)	2643202/97050439
24	(15, 8, 1)	687439/38273329
24	(15, 7, 2)	387392/39075271
24	(13, 7, 4)	2224115/77459599
24	(12, 10, 2)	1469825/87141847

Delete those four columns from A . The remaining square submatrix is B_f . Use the prime

$$p = 1000000000000000031$$

and the two roots of $X^2 - 5$ modulo p ,

$$\rho_+ = 968101896520624317, \quad \rho_- = 31898103479375714.$$

They satisfy $\rho_{\pm}^2 \equiv 5 \pmod{p}$ and $\rho_+ + \rho_- \equiv 0 \pmod{p}$. All denominators occurring in the selected submatrices are invertible modulo p . Exact modular elimination gives

	$\sqrt{5} \mapsto \rho_+$	$\sqrt{5} \mapsto \rho_-$
$f = 23$	650097745717062046	139778200766197651
$f = 24$	550949013298233594	966277299582556251

for $\det B_f$ modulo p .

Proposition 2 (Full row rank). $\text{rank}_{\mathbb{Q}(\sqrt{5})} A_{23} = 52$ and $\text{rank}_{\mathbb{Q}(\sqrt{5})} A_{24} = 57$.

Proof. If $\det B_f$ were zero in $\mathbb{Q}(\sqrt{5})$, every defined reduction of it would be zero. The displayed residues are nonzero, so $\det B_f \neq 0$. Hence the rank is at least the number of rows; it cannot exceed that number. The affine solution spaces therefore have dimensions $56 - 52 = 4$ and $61 - 57 = 4$. $\square$

The original search used modular solutions in both embeddings, Chinese remainder accumulation over 500 good primes, and rational reconstruction with an accumulated modulus of roughly 9001 decimal digits per frequency. That search is not a premise of the theorem: the reconstructed coordinates are accepted only after direct exact substitution into $AW = b$.

17 The complete exact certificate

The exact orbit mass has the form

$$W_{\lambda} = a_{\lambda} + b_{\lambda}\sqrt{5}, \quad a_{\lambda}, b_{\lambda} \in \mathbb{Q}.$$

Every numerator and denominator for all $56 + 61 = 117$ orbit masses appears in the embedded JSON file `ClassI_Cubature_f23_f24_exact_certificate_from_tex_v1.1.json`, created directly from the

`filecontents*` block at the beginning of this LaTeX source. It is a whitespace-reformatted, JSON-semantically identical copy of archived certificate v1.1; no numeric string or data value is changed.

The canonical exact weight-text hashes are

$$\begin{aligned} f = 23 : & \text{ f56551614718e7fd45de5a95cd01767a0060904b96ae2a42c13f99b165c0f9f7,} \\ f = 24 : & \text{ 86a96d2c91fffa5cbe2382299eb31353635d3d20396a61ae40a03bfd163f2d27.} \end{aligned}$$

These hashes are computed from, in orbit order, the exact canonical lines

$$a, b, c : a \text{Numerator} / a \text{Denominator} : b \text{Numerator} / b \text{Denominator}.$$

Proposition 3 (Exact certificate identities). *For the 117 embedded masses:*

1. every mass is positive in the real embedding $\sqrt{5} > 0$;
2. the mass sum is exactly $1 + 0\sqrt{5}$ at each frequency;
3. all 52 frequency-23 moment residuals are $0 + 0\sqrt{5}$;
4. all 57 frequency-24 moment residuals are $0 + 0\sqrt{5}$.

Proof. The embedded verifier represents each field element as a pair of Python `Fraction` objects and implements the identities

$$(a + b\sqrt{5})(c + d\sqrt{5}) = (ac + 5bd) + (ad + bc)\sqrt{5},$$

$$(a + b\sqrt{5})^{-1} = \frac{a - b\sqrt{5}}{a^2 - 5b^2}.$$

It rebuilds V , the Class-I representatives, U_6 , U_{10} , every exact spherical moment, and both matrices A from the definitions in this dossier. It then parses all 117 coefficient pairs, applies the exact sign test above, checks both exact sums, and directly evaluates every row of $AW - b$. An equality assertion succeeds only when both rational components are exactly zero. It also recomputes the determinant witnesses and canonical weight hashes. The independently rerun output on 5 September 2026 is:

```
PASS f=23 tau=71 nodes=5292 orbits=56 rows=52 rank=52 free=4 min=(21, 1, 1) ~
↪ 0.003777725693780652542084082262449790124026578124021415259901
PASS f=24 tau=75 nodes=5762 orbits=61 rows=57 rank=57 free=4 min=(23, 1, 0) ~
↪ 0.003412253320984930573099634686301743266126075350598455734277
ALL PASS: 109 exact invariant moment identities; strict positivity and rank certified over
↪ Q(sqrt(5)).
certificate_sha256 e886510ec03afe014ad143d6d0b45677fc2bb58a7df495374b5780cf79d5b5af
```

The decimals printed after the minima are not used by any equality or sign assertion. The complete exact row ledger is embedded as `ClassI_Cubature_f23_f24_verification_ledger_v1.1.txt`. $\square$

Proof of the two main theorems. The preceding proposition gives $W_\lambda > 0$, exact normalization, and $AW = b$ for every row in $\mathcal{I}_{71}$ and $\mathcal{I}_{75}$. By invariant reduction, the resulting H_3 -invariant rules integrate every spherical polynomial through the respective target degrees. Dividing each positive orbit mass by its positive orbit cardinality yields a positive weight at every node. This proves the two stated existence claims. $\square$

18 Exact minima and the binary64 correction

Minimum selection is performed by exact signs of all pairwise differences in $\mathbb{Q}(\sqrt{5})$. The certified results are

f	minimum orbit	minimum total orbit mass (60 decimal places)
23	(21, 1, 1)	0.003777725693780652542084082262449790124026578124021415259901
24	(23, 1, 0)	0.003412253320984930573099634686301743266126075350598455734277

For $f = 23$, the second-smallest orbit is (23, 0, 0) and the exact-positive gap is approximately 0.0000000000000016097086588899275533972958682615348747500785135. For $f = 24$, the second-smallest orbit is (16, 8, 0) and the gap is approximately 0.0000000000000682670223110140296092510846004654888220013042813.

Candidate v1.0 rendered $a + b\sqrt{5}$ as `float(a)+float(b)*sqrt(5)` even when a and b were enormous and nearly cancelled. It therefore printed the incorrect minima 0.003777725680265576 and 0.003412253223359585 and used binary64 to select the minimum. Version 1.1 replaces that step with exact comparisons and uses high-precision decimal arithmetic only for display. The exact weight vectors, moment identities, positivity decisions, rank witnesses, and canonical weight-vector hashes were unchanged.

19 Reproduction from this one LaTeX source

Compiling this source emits the complete semantic certificate, exact verifier, ledger, audit, archived transcript, and a small launcher. Run

```
python3 extract_and_verify_ClassI_f23_f24_from_tex_v1.1.py
```

in the compilation directory. The expected output is embedded as `ClassI_Cubature_f23_f24_expected_transcript_from_tex_v1.1.txt`.

The archived artifact identities are:

artifact	SHA-256
archived exact certificate v1.1	e886510ec03afe014ad143d6d0b45677fc2bb58a7df495374b5780cf79d5b5af
verifier v1.1	41d83727473850ebf0bbea0f57f38a64c08d9849170a880b23163769509ce1
verification ledger v1.1	4d56dc2c65466fc89e6c275fdc38012dbca2c5b74456cf045e7ceedb30a38e6e
LaTeX-emitted semantic JSON copy	3ecbf6656eee22044f1a9cdaf84e9a3e7365622dcd4d4a1734a1b7c83e15d72b

The original certificate is one-line canonical JSON. The LaTeX-emitted copy adds whitespace only so that no input line exceeds TeX's buffer; JSON parsing returns the same object and every exact rational string is identical. The verifier's two canonical weight-vector hashes make that identity independently checkable at the mathematical-data level.

20 Claim boundary

The proof establishes exactly these statements:

X_{23} admits a strictly positive H_3 -invariant rule through degree 71,

X_{24} admits a strictly positive H_3 -invariant rule through degree 75.

It does not establish maximality at either frequency, an all-frequency positivity theorem, independent clean-room reproduction, peer review, or any logical dependence on Ω_0 or SRSD.

A Complete orbit order and exact sign results

Every exact coefficient pair itself is in the embedded JSON certificate. The following tables record the complete canonical orbit order and the exact sign result for every mass.

A.1 $f = 23$

index	orbit	sign	designation
0	(23, 0, 0)	+	–
1	(22, 1, 0)	+	–
2	(21, 2, 0)	+	–
3	(21, 1, 1)	+	minimum
4	(20, 3, 0)	+	–
5	(20, 2, 1)	+	–
6	(19, 4, 0)	+	–
7	(19, 3, 1)	+	–
8	(19, 2, 2)	+	–
9	(18, 5, 0)	+	–
10	(18, 4, 1)	+	–
11	(18, 3, 2)	+	–
12	(17, 6, 0)	+	–
13	(17, 5, 1)	+	free
14	(17, 4, 2)	+	–
15	(17, 3, 3)	+	–
16	(16, 7, 0)	+	–
17	(16, 6, 1)	+	free
18	(16, 5, 2)	+	–
19	(16, 4, 3)	+	–
20	(15, 8, 0)	+	–
21	(15, 7, 1)	+	–
22	(15, 6, 2)	+	–
23	(15, 5, 3)	+	–
24	(15, 4, 4)	+	–
25	(14, 9, 0)	+	–
26	(14, 8, 1)	+	–
27	(14, 7, 2)	+	–

index	orbit	sign	designation
28	(14, 6, 3)	+	–
29	(14, 5, 4)	+	–
30	(13, 10, 0)	+	–
31	(13, 9, 1)	+	free
32	(13, 8, 2)	+	–
33	(13, 7, 3)	+	–
34	(13, 6, 4)	+	–
35	(13, 5, 5)	+	–
36	(12, 11, 0)	+	–
37	(12, 10, 1)	+	–
38	(12, 9, 2)	+	–
39	(12, 8, 3)	+	–
40	(12, 7, 4)	+	–
41	(12, 6, 5)	+	–
42	(11, 11, 1)	+	–
43	(11, 10, 2)	+	–
44	(11, 9, 3)	+	–
45	(11, 8, 4)	+	–
46	(11, 7, 5)	+	–
47	(11, 6, 6)	+	–
48	(10, 10, 3)	+	–
49	(10, 9, 4)	+	–
50	(10, 8, 5)	+	free
51	(10, 7, 6)	+	–
52	(9, 9, 5)	+	–
53	(9, 8, 6)	+	–
54	(9, 7, 7)	+	–
55	(8, 8, 7)	+	–

A.2 $f = 24$

index	orbit	sign	designation
0	(24, 0, 0)	+	–
1	(23, 1, 0)	+	minimum
2	(22, 2, 0)	+	–
3	(22, 1, 1)	+	–
4	(21, 3, 0)	+	–
5	(21, 2, 1)	+	–
6	(20, 4, 0)	+	–
7	(20, 3, 1)	+	–
8	(20, 2, 2)	+	–
9	(19, 5, 0)	+	–
10	(19, 4, 1)	+	–
11	(19, 3, 2)	+	–
12	(18, 6, 0)	+	–
13	(18, 5, 1)	+	–

index	orbit	sign	designation
14	(18, 4, 2)	+	—
15	(18, 3, 3)	+	—
16	(17, 7, 0)	+	—
17	(17, 6, 1)	+	—
18	(17, 5, 2)	+	—
19	(17, 4, 3)	+	—
20	(16, 8, 0)	+	—
21	(16, 7, 1)	+	—
22	(16, 6, 2)	+	—
23	(16, 5, 3)	+	—
24	(16, 4, 4)	+	—
25	(15, 9, 0)	+	—
26	(15, 8, 1)	+	free
27	(15, 7, 2)	+	free
28	(15, 6, 3)	+	—
29	(15, 5, 4)	+	—
30	(14, 10, 0)	+	—
31	(14, 9, 1)	+	—
32	(14, 8, 2)	+	—
33	(14, 7, 3)	+	—
34	(14, 6, 4)	+	—
35	(14, 5, 5)	+	—
36	(13, 11, 0)	+	—
37	(13, 10, 1)	+	—
38	(13, 9, 2)	+	—
39	(13, 8, 3)	+	—
40	(13, 7, 4)	+	free
41	(13, 6, 5)	+	—
42	(12, 12, 0)	+	—
43	(12, 11, 1)	+	—
44	(12, 10, 2)	+	free
45	(12, 9, 3)	+	—
46	(12, 8, 4)	+	—
47	(12, 7, 5)	+	—
48	(12, 6, 6)	+	—
49	(11, 11, 2)	+	—
50	(11, 10, 3)	+	—
51	(11, 9, 4)	+	—
52	(11, 8, 5)	+	—
53	(11, 7, 6)	+	—
54	(10, 10, 4)	+	—
55	(10, 9, 5)	+	—
56	(10, 8, 6)	+	—
57	(10, 7, 7)	+	—
58	(9, 9, 6)	+	—
59	(9, 8, 7)	+	—
60	(8, 8, 8)	+	—

B Complete exact moment-residual index

Each residual is displayed as its exact coefficient pair (a, b) in $a + b\sqrt{5}$. Thus $(0, 0)$ means an exact zero in $\mathbb{Q}(\sqrt{5})$, not a rounded floating-point residual.

B.1 $f = 23$, target degree 71

index	invariant degree	(r, s)	residual (a, b)
0	0	(0, 0)	(0, 0)
1	6	(1, 0)	(0, 0)
2	10	(0, 1)	(0, 0)
3	12	(2, 0)	(0, 0)
4	16	(1, 1)	(0, 0)
5	18	(3, 0)	(0, 0)
6	20	(0, 2)	(0, 0)
7	22	(2, 1)	(0, 0)
8	24	(4, 0)	(0, 0)
9	26	(1, 2)	(0, 0)
10	28	(3, 1)	(0, 0)
11	30	(5, 0)	(0, 0)
12	30	(0, 3)	(0, 0)
13	32	(2, 2)	(0, 0)
14	34	(4, 1)	(0, 0)
15	36	(6, 0)	(0, 0)
16	36	(1, 3)	(0, 0)
17	38	(3, 2)	(0, 0)
18	40	(5, 1)	(0, 0)
19	40	(0, 4)	(0, 0)
20	42	(7, 0)	(0, 0)
21	42	(2, 3)	(0, 0)
22	44	(4, 2)	(0, 0)
23	46	(6, 1)	(0, 0)
24	46	(1, 4)	(0, 0)
25	48	(8, 0)	(0, 0)
26	48	(3, 3)	(0, 0)
27	50	(5, 2)	(0, 0)
28	50	(0, 5)	(0, 0)
29	52	(7, 1)	(0, 0)
30	52	(2, 4)	(0, 0)
31	54	(9, 0)	(0, 0)
32	54	(4, 3)	(0, 0)
33	56	(6, 2)	(0, 0)
34	56	(1, 5)	(0, 0)
35	58	(8, 1)	(0, 0)
36	58	(3, 4)	(0, 0)
37	60	(10, 0)	(0, 0)
38	60	(5, 3)	(0, 0)
39	60	(0, 6)	(0, 0)

index	invariant degree	(r, s)	residual (a, b)
40	62	$(7, 2)$	$(0, 0)$
41	62	$(2, 5)$	$(0, 0)$
42	64	$(9, 1)$	$(0, 0)$
43	64	$(4, 4)$	$(0, 0)$
44	66	$(11, 0)$	$(0, 0)$
45	66	$(6, 3)$	$(0, 0)$
46	66	$(1, 6)$	$(0, 0)$
47	68	$(8, 2)$	$(0, 0)$
48	68	$(3, 5)$	$(0, 0)$
49	70	$(10, 1)$	$(0, 0)$
50	70	$(5, 4)$	$(0, 0)$
51	70	$(0, 7)$	$(0, 0)$

B.2 $f = 24$, target degree 75

index	invariant degree	(r, s)	residual (a, b)
0	0	$(0, 0)$	$(0, 0)$
1	6	$(1, 0)$	$(0, 0)$
2	10	$(0, 1)$	$(0, 0)$
3	12	$(2, 0)$	$(0, 0)$
4	16	$(1, 1)$	$(0, 0)$
5	18	$(3, 0)$	$(0, 0)$
6	20	$(0, 2)$	$(0, 0)$
7	22	$(2, 1)$	$(0, 0)$
8	24	$(4, 0)$	$(0, 0)$
9	26	$(1, 2)$	$(0, 0)$
10	28	$(3, 1)$	$(0, 0)$
11	30	$(5, 0)$	$(0, 0)$
12	30	$(0, 3)$	$(0, 0)$
13	32	$(2, 2)$	$(0, 0)$
14	34	$(4, 1)$	$(0, 0)$
15	36	$(6, 0)$	$(0, 0)$
16	36	$(1, 3)$	$(0, 0)$
17	38	$(3, 2)$	$(0, 0)$
18	40	$(5, 1)$	$(0, 0)$
19	40	$(0, 4)$	$(0, 0)$
20	42	$(7, 0)$	$(0, 0)$
21	42	$(2, 3)$	$(0, 0)$
22	44	$(4, 2)$	$(0, 0)$
23	46	$(6, 1)$	$(0, 0)$
24	46	$(1, 4)$	$(0, 0)$
25	48	$(8, 0)$	$(0, 0)$
26	48	$(3, 3)$	$(0, 0)$
27	50	$(5, 2)$	$(0, 0)$
28	50	$(0, 5)$	$(0, 0)$
29	52	$(7, 1)$	$(0, 0)$

index	invariant degree	(r, s)	residual (a, b)
30	52	(2, 4)	(0, 0)
31	54	(9, 0)	(0, 0)
32	54	(4, 3)	(0, 0)
33	56	(6, 2)	(0, 0)
34	56	(1, 5)	(0, 0)
35	58	(8, 1)	(0, 0)
36	58	(3, 4)	(0, 0)
37	60	(10, 0)	(0, 0)
38	60	(5, 3)	(0, 0)
39	60	(0, 6)	(0, 0)
40	62	(7, 2)	(0, 0)
41	62	(2, 5)	(0, 0)
42	64	(9, 1)	(0, 0)
43	64	(4, 4)	(0, 0)
44	66	(11, 0)	(0, 0)
45	66	(6, 3)	(0, 0)
46	66	(1, 6)	(0, 0)
47	68	(8, 2)	(0, 0)
48	68	(3, 5)	(0, 0)
49	70	(10, 1)	(0, 0)
50	70	(5, 4)	(0, 0)
51	70	(0, 7)	(0, 0)
52	72	(12, 0)	(0, 0)
53	72	(7, 3)	(0, 0)
54	72	(2, 6)	(0, 0)
55	74	(9, 2)	(0, 0)
56	74	(4, 5)	(0, 0)

C Embedded machine-proof inventory

The opening `filecontents*` blocks are part of this LaTeX source and contain, without external downloads:

1. all 117 exact values $a_\lambda + b_\lambda\sqrt{5}$;
2. the complete exact-arithmetic verifier;
3. the complete 109-row equality ledger and all 117 sign outcomes;
4. the correction audit and archived passing transcript;
5. a launcher that checks the emitted certificate bytes and runs the verifier.

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