

OMEGA and Navier–Stokes

Geometric Transport and Exact Invariants

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Abstract

The supplied OMEGA geometry and golden quadratic form admit an explicit embedding in incompressible three-dimensional fluid motion. A two-parameter family of volume-preserving deformations transports the golden norm and preserves the cubic quantity $z(x^2 + xy - y^2)$. Its similarity subgroup transports the entire marked planar geometry while keeping $(G+C)/a = \varphi + \pi$ constant. Its anisotropic subgroup realizes the square of the normalized Fibonacci matrix and its scalar contraction observation. Smooth time paths have explicit affine Navier–Stokes realizations. A curl localization gives finite-energy, globally smooth forced realizations, starting from rest, for every smooth finite path in this deformation family, with positive scale, identity initial map, and constant schedules on neighborhoods of the initial and final times, acting on a compact marked set. Further identities connect the construction to the frozen residual encoder, Sacks transport, fluctuation stresses, and spherical cubature.

Scope and contribution. This paper proves an explicit realization and preservation theorem. The normalized readout is fixed by the marked geometry; the fluid construction preserves that readout on its similarity branch. All analytic arguments needed for the fluid theorem are given below. The supplied Lean and Julia packages support specified geometric and arithmetic interfaces; their relation to the new results is documented in Section 11. The construction belongs to the established method of prescribing a field and computing its forcing residual, often used in manufactured solutions [4]. Curl-based divergence-free constructions also occur in that literature [5]. The contribution developed here is the explicit integration of the specified marked geometry, quadratic algebra, transport and arithmetic interfaces. No priority claim is made for the underlying affine-flow or cutoff techniques. No numerical PDE solver or experimental flow is validated by the symbolic calculations in this paper.

Conventions. SRSD v1.0 is used as an external exact encoding interface. Physical coordinates retain their units; scalar observations and spiral displays are explicitly nondimensionalized. All initial-to-current particle maps are normalized at the initial time. The Navier–Stokes equation uses density one.

Organization. The geometric measurements and deformation are established first, followed by their affine and finite-energy fluid realizations. Subsequent sections prove the connections to scalar observations, integer ledgers, spiral coordinates, stress and cubature. The final sections record source integration and reproducibility; Appendix A gives the exact floor-boundary certificate used in the arithmetic connection.

1 Definitions and the geometric question

Write

$$\varphi = \frac{1 + \sqrt{5}}{2}, \quad \varphi' = \frac{1 - \sqrt{5}}{2}, \quad \Omega_0 = \varphi + \pi, \quad \tau_\Omega = \Omega_0^{-1}, \quad \lambda = \tau_\Omega \varphi.$$

The symbol τ_Ω denotes a constant, not the remaining-time variable used in some fluid blowup constructions. For the marked planar geometry defined below, scale $a > 0$ and selected lengths $G = a\varphi$, $C = a\pi$ give the normalized readout

$$\mathcal{I} = \frac{G + C}{a} = \Omega_0. \quad (1)$$

The selection rule is part of the specification. Its numerical evaluation follows from the construction, as the next proposition proves. The problem is to transport that marked construction by an incompressible fluid velocity while preserving the readout and connecting the transport to the golden algebra.

Put $\varepsilon = \pi/648000$, $\delta = \varepsilon/2$, $s_\delta = \sin \delta$, and $c_\delta = \cos \delta$. To distinguish the seed cosine from the deformation scale, only the latter will be denoted by $c(t)$. For $a > 0$, define planar points and a circle parametrization by

$$m_a(h) = (ac_\delta, ah), \quad q_a(\theta) = a(\cos \theta, \sin \theta).$$

The six labeled heights are

$$-(\varphi^2 + s_\delta), \quad -(1 + s_\delta), \quad -s_\delta, \quad s_\delta, \quad 1 + s_\delta, \quad \varphi^2 + s_\delta.$$

Define the compact marked outline

$$\begin{aligned} \Gamma_\Omega(a) &= q_a([\delta, 2\pi - \delta]) \\ &\cup m_a([s_\delta, \varphi^2 + s_\delta]) \cup m_a([- \varphi^2 - s_\delta, -s_\delta]), \end{aligned}$$

with its six labels and the opposite semicircle retained as marked subsets. Let G be the mean of the two outer straight lengths, from levels $\pm(1 + s_\delta)$ to $\pm(\varphi^2 + s_\delta)$, and let C be the arc length of $q_a([\pi/2, 3\pi/2])$.

Proposition 1.1 (Selected measurements of the marked construction). *The marked pieces used by G and C lie in $\Gamma_\Omega(a)$, and*

$$G = a\varphi, \quad C = a\pi, \quad \frac{G + C}{a} = \Omega_0, \quad \frac{G}{G + C} = \lambda.$$

The total component length of the outline is $a(2\pi - \varepsilon) + 2a\varphi^2$, which is strictly greater than $G + C$. A rigid planar placement preserves these statements.

Proof. Since $0 < \delta < \pi/2$, the opposite semicircle is retained. Each outer segment lies inside the corresponding extension. For all real h, k , $|m_a(h) - m_a(k)| = a|h - k|$. The outer height difference is $\varphi^2 - 1 = \varphi$, giving both outer lengths $a\varphi$ and hence $G = a\varphi$. Also $|q'_a(\theta)| = a$, so the opposite semicircle has length $a\pi$. Cancelling a proves the ratios. The retained arc has length $a(2\pi - 2\delta)$ and each full extension has length $a\varphi^2$. Subtracting $G + C$ from their sum gives $a(\pi - \varepsilon + \varphi + 2) > 0$. Rigid placements preserve lengths. $\square$

Thus $G + C$ is a selected measurement, not the total outline length. The algebraic specification of these marks, rather than an unlabeled silhouette, is the geometric input used throughout this paper.

Use orthonormal spatial coordinates $X = (x, y, z) = (w, z)$, $w \in \mathbb{R}^2$, with the initial marked construction in $z = 0$. A fixed rigid change of coordinates places any supplied planar OMEGA realization in this convention. Define

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \quad H = \begin{pmatrix} 1 & 1/2 \\ 1/2 & -1 \end{pmatrix}, \quad K = \frac{2M - I_2}{\sqrt{5}} = \frac{1}{\sqrt{5}} \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix}.$$

The golden quadratic form and its linear observations are

$$\mathcal{N}(w) = w^T H w = x^2 + xy - y^2, \quad \ell_+(w) = x + \varphi y, \quad \ell_-(w) = x + \varphi' y.$$

As in the supplied dossier, $\mathcal{N}$ is indefinite; it is not kinetic energy or a positive metric norm. Direct multiplication gives

$$K^T = K, \quad \text{tr } K = 0, \quad K^2 = I_2, \quad K^T H + H K = 0, \quad \ell_+ K = \ell_+, \quad \ell_- K = -\ell_-. \quad (2)$$

Here the last two identities treat the observations as row vectors.

2 A volume-preserving geometric family

Theorem 2.1 (Golden deformation and its cubic invariant). *For $c > 0$ and $\eta \in \mathbb{R}$, define*

$$\Phi_{c,\eta}(w, z) = (ce^{\eta K} w, c^{-2} z). \quad (3)$$

Then

$$\Phi_{c,\eta} \circ \Phi_{d,\theta} = \Phi_{cd,\eta+\theta}, \quad \det D\Phi_{c,\eta} = 1, \quad (4)$$

$$\mathcal{N}(ce^{\eta K} w) = c^2 \mathcal{N}(w), \quad \ell_+(ce^{\eta K} w) = ce^{\eta} \ell_+(w). \quad (5)$$

In particular,

$$\boxed{J(X) = z\mathcal{N}(w) = z(x^2 + xy - y^2), \quad J \circ \Phi_{c,\eta} = J.} \quad (6)$$

Proof. Since all exponentials are generated by the same matrix, their products add parameters. Also $\det e^{\eta K} = e^{\eta \text{tr } K} = 1$, so the planar determinant is c^2 and the axial factor c^{-2} compensates it. Differentiating $e^{\eta K^T} H e^{\eta K}$ and applying (2) gives zero, hence that expression equals H . This proves the norm formula. The eigen-observation identities imply $\ell_+ e^{\eta K} = e^{\eta} \ell_+$ and similarly $\ell_- e^{\eta K} = e^{-\eta} \ell_-$. Finally, multiply $c^2 \mathcal{N}(w)$ by $c^{-2} z$. $\square$

Remark 2.2. The invariant J concerns particle position. It is a first integral for the flow constructed below, not a claim that every Navier–Stokes velocity preserves this cubic function. On the geometry plane $z = 0$, $J = 0$; nonzero levels describe additional three-dimensional trajectories.

Proposition 2.3 (Why compensation is necessary). *A differentiable incompressible particle flow has Jacobian determinant one when initialized at the identity. A uniform three-dimensional homothety $X \mapsto cX$ can therefore be such a flow map only when $c = 1$. Planar scaling by c can be realized by compensating its area factor with the axial factor c^{-2} in (3).*

Proof. Jacobi's formula for a flow map Ψ_t gives $\frac{d}{dt} \det D\Psi_t = (\text{div } u) \circ \Psi_t \det D\Psi_t = 0$. The determinant of cI_3 is c^3 , whereas $\det \text{diag}(c, c, c^{-2}) = 1$. $\square$

2.1 A real-parameter family and the role of the golden choice

Proposition 2.4 (The algebra does not single out the golden parameter). *For every $\kappa \in \mathbb{R}$, put*

$$M_\kappa = \begin{pmatrix} 0 & 1 \\ 1 & \kappa \end{pmatrix}, \quad H_\kappa = \begin{pmatrix} 1 & \kappa/2 \\ \kappa/2 & -1 \end{pmatrix}, \quad K_\kappa = \frac{2M_\kappa - \kappa I}{\sqrt{\kappa^2 + 4}}, \quad \alpha_\kappa = \frac{\kappa + \sqrt{\kappa^2 + 4}}{2} > 0.$$

Then K_κ is symmetric and traceless, $K_\kappa^2 = I$, and

$$K_\kappa^T H_\kappa + H_\kappa K_\kappa = 0, \quad M_\kappa^T H_\kappa M_\kappa = -H_\kappa, \quad e^{2 \log(\alpha_\kappa) K_\kappa} = M_\kappa^2.$$

The maps $(w, z) \mapsto (ce^{\eta K_\kappa} w, c^{-2} z)$ preserve volume and $z(x^2 + \kappa xy - y^2)$. The choice $\kappa = 1$ gives the golden family.

Proof. Matrix multiplication proves the first five identities. The eigenvalues of M_κ are α_κ and $-\alpha_\kappa^{-1}$, since its characteristic polynomial is $t^2 - \kappa t - 1$. On the corresponding eigenspaces K_κ has eigenvalues 1 and -1 . Hence both sides of the exponential identity have eigenvalues α_κ^2 and α_κ^{-2} on these same complementary spaces. Tracelessness gives the determinant statement. Differentiating $e^{\eta K_\kappa^T} H_\kappa e^{\eta K_\kappa}$ gives zero, and multiplying the resulting planar factor c^2 by the axial factor c^{-2} proves the cubic invariance. Also $\sqrt{\kappa^2 + 4} > |\kappa|$, proving positivity. $\square$

This generalization identifies exactly where the golden choice enters: it is supplied by the marked geometry and recurrence. Incompressibility and the affine fluid equation do not force $\kappa = 1$.

3 Exact Navier–Stokes realization

We use density one and the Euclidean equations

$$\partial_t u + (u \cdot \nabla) u - \nu \Delta u + \nabla p = f, \quad \operatorname{div} u = 0, \quad \nu > 0. \quad (7)$$

Pressure is pressure divided by density. Spatial coordinates have length units; the rates below have inverse-time units.

Theorem 3.1 (Affine realization for every positive viscosity). *Let $c(t) > 0$ and $\eta(t)$ be smooth, and set*

$$\gamma = \dot{c}/c, \quad \beta = \dot{\eta}, \quad B(t) = \begin{pmatrix} \gamma I_2 + \beta K & 0 \\ 0 & -2\gamma \end{pmatrix}.$$

Then

$$\boxed{u_{\text{aff}}(X, t) = B(t)X, \quad p_{\text{aff}}(X, t) = -\frac{1}{2}X^T(\dot{B} + B^2)X} \quad (8)$$

solve (7) with $f = 0$ for every $\nu > 0$. For any initial time t_0 , their initial-to-current particle map is

$$\boxed{\Psi_{t,t_0} = \Phi_{c(t)/c(t_0), \eta(t)-\eta(t_0)}} \quad (9)$$

In particular, if $c(0) = 1$ and $\eta(0) = 0$, this is $\Phi_{c(t), \eta(t)}$. Along every particle trajectory, J in (6) is constant.

Proof. B is symmetric and traceless. Therefore

$$\operatorname{div}(BX) = \operatorname{tr} B = 0, \quad \Delta(BX) = 0, \quad \partial_t(BX) = \dot{B}X, \quad ((BX) \cdot \nabla)(BX) = B^2X.$$

Both $\dot{B}$ and B^2 are symmetric, so $\nabla p_{\text{aff}} = -(\dot{B} + B^2)X$. Substitution proves the entire momentum equation, including the viscous term. Differentiation gives $\partial_t \Psi_{t,t_0} = B(t)\Psi_{t,t_0}$ and $\Psi_{t_0,t_0} = I_3$. Uniqueness for this linear ODE proves (9). The first integral follows from Theorem 2.1; equivalently, $(BX) \cdot \nabla J = 0$. $\square$

The affine velocity is irrotational because B is symmetric. Except when zero, it has infinite total kinetic energy on $\mathbb{R}^3$. These are exact local and whole-space algebraic solutions, not finite-energy whole-space solutions. Section 6 supplies the finite-energy forced realization. In that realization the viscous momentum term can be nonzero in the transition region while it vanishes in the affine core; the core still has strain and viscous dissipation as detailed in Proposition 6.4.

The mechanism can be read in a fixed orthonormal eigenbasis. If $O^T K O = \text{diag}(1, -1)$, then in coordinates $(O^T w, z)$ the deformation is $\text{diag}(ce^\eta, ce^{-\eta}, c^{-2})$: three compensating stretches. The same momentum calculation works for any smooth symmetric trace-free matrix $B(t)$. For a noncommuting family of such matrices, its particle map is the fundamental matrix of the linear ODE and need not have the commuting formula used here.

4 The OMEGA shape and scalar contraction inside the flow

Corollary 4.1 (A material OMEGA invariant). *Use $\eta(t) = 0$ and normalize $c(0) = 1$. Any compact marked planar OMEGA construction $\Gamma_\Omega(a_0)$ placed in $z = 0$ is carried by the particle map to $\Gamma_\Omega(a_0 c(t))$. Consequently*

$$a(t) = a_0 c(t), \quad G(t) = a_0 c(t) \varphi, \quad C(t) = a_0 c(t) \pi, \quad \boxed{\frac{G(t) + C(t)}{a(t)} = \Omega_0}.$$

Angles, marked length ratios, and the selected circular and golden features are preserved. For any dimensionless real linear planar observable L , define the three-dimensional observation $\tilde{V}(w, z) = 1 + L(w)$. It obeys

$$\tilde{V}(\Phi_{c,0}(w, z)) = 1 + c(\tilde{V}(w, z) - 1). \quad (10)$$

At $c = \lambda$ this is the supplied map $T(v) = 1 + \lambda(v - 1)$.

Proof. On $z = 0$ the map is the Euclidean similarity $w \mapsto cw$. Every length scales by c and every angle remains the same. Substitute the initial measurement identities and use linearity of L . $\square$

For example, setting $c(t) = \lambda^{t/t_0}$ on a finite interval realizes one contraction step every duration $t_0 > 0$. To start and end at rest, replace t/t_0 by a smooth schedule $\sigma(t)$, zero near the initial time and constant near the final time. The exact observation becomes $1 + \lambda^{\sigma(t)}(V_0 - 1)$. There is no imposed boundary along the material curve: it is a marked set of fluid particles, not a rigid obstacle or a wall.

5 The normalized Fibonacci matrix inside the same family

Let $A_\Omega = \tau_\Omega M$, as in the supplied golden-norm dossier. The spectral projectors of K are $P_\pm = (I_2 \pm K)/2$, so

$$e^{\eta K} = e^\eta P_+ + e^{-\eta} P_-.$$

The eigenvalues of M are φ and $\varphi' = -\varphi^{-1}$. It follows that

$$e^{2(\log \varphi)K} = \varphi^2 P_+ + \varphi^{-2} P_- = M^2. \quad (11)$$

Corollary 5.1 (A continuous fluid realization of two matrix steps). *Choose a smooth dimensionless schedule $\sigma(t)$ with $\sigma(0) = 0$ and set*

$$c(t) = \tau_\Omega^{2\sigma(t)}, \quad \eta(t) = 2\sigma(t) \log \varphi.$$

At $\sigma = 1$, the material map is

$$\Phi(w, z) = (A_\Omega^2 w, \tau_\Omega^{-4} z). \quad (12)$$

Fix a reference length $a_{\text{ref}} > 0$ and define $\tilde{V}_+(w, z) = 1 + \ell_+(w)/a_{\text{ref}}$. Writing $V_+(t) = \tilde{V}_+(w(t), z(t))$, the entire path satisfies

$$V_+(t) = 1 + \lambda^{2\sigma(t)}(V_+(0) - 1), \quad \mathcal{N}(w(t)) = \tau_\Omega^{4\sigma(t)}\mathcal{N}(w(0)), \quad z(t)\mathcal{N}(w(t)) = z(0)\mathcal{N}(w(0)).$$

Proof. Apply (11) and (5)–(6); $ce^\eta = (\tau_\Omega\varphi)^{2\sigma} = \lambda^{2\sigma}$. $\square$

For an arbitrary initial schedule, put $\Delta\sigma(t) = \sigma(t) - \sigma(0)$. Formula (9) gives

$$V_+(t) = 1 + \lambda^{2\Delta\sigma(t)}(V_+(0) - 1), \quad \mathcal{N}(w(t)) = \tau_\Omega^{4\Delta\sigma(t)}\mathcal{N}(w(0)).$$

Two matrix steps then occur when $\Delta\sigma = 1$. A constant schedule produces the identity particle map, as these formulas require.

The square is intentional: $\det A_\Omega = -\tau_\Omega^2 < 0$, whereas a smooth planar flow map has positive determinant. Our plane-invariant flow realizes A_Ω^2 . A single A_Ω remains the original discrete orientation-reversing map.

The similarity case $\eta = 0$ preserves the physical shape and its Euclidean length ratios. The anisotropic case generally turns circles into ellipses, and does not preserve that same Euclidean shape. Their unity is the explicitly defined family (3), not an assertion that these two actions coincide. When $c = 1$, $\eta = 2 \log \varphi$, the family instead realizes M^2 on the plane and preserves $\mathcal{N}$ itself.

6 Finite energy and smooth compact forcing

Theorem 6.1 (Localized realization of a prescribed finite geometric motion). *Let $T > 0$, let $c, \eta \in C^\infty(\mathbb{R})$ with $c > 0$, and suppose $c = 1$, $\eta = 0$ near and before $t = 0$, while c, η are constant near and after $t = T$. Let $\mathcal{K} \subset \mathbb{R}^3$ be compact. There exist globally smooth fields u, p , independent of viscosity, such that for every $\nu > 0$ there is a force $f = f_\nu \in C_c^\infty(\mathbb{R}^3 \times (0, \infty); \mathbb{R}^3)$ satisfying (7) and the following conditions:*

1. $u(\cdot, 0) = 0$, and u, p have compact spatial support and vanish outside a compact subinterval of $(0, T)$;
2. $\sup_{t \geq 0} \int_{\mathbb{R}^3} |u|^2 dX < \infty$;
3. each initial point $X_0 \in \mathcal{K}$ follows exactly $X(t) = \Phi_{c(t), \eta(t)}(X_0)$;
4. f vanishes in a neighborhood of the complete marked trajectory set; the forcing is confined to a surrounding transition region.

Consequently, Corollaries 4.1 and 5.1 hold for these marked particles in finite-energy, smoothly forced Navier–Stokes solutions.

Proof. The set $\mathcal{T} = \{\Phi_{c(t), \eta(t)}(X_0) : X_0 \in \mathcal{K}, 0 \leq t \leq T\}$ is compact by continuity. Choose a smooth spatial cutoff χ with compact support, equal to one on an open neighborhood of $\mathcal{T}$. For the affine fields of Theorem 3.1, define

$$\mathcal{A}(X, t) = \frac{1}{3}(u_{\text{aff}}(X, t) \times X), \quad u = \text{curl}(\chi\mathcal{A}), \quad p = \chi p_{\text{aff}}. \quad (13)$$

The vector identity for the curl of a cross product gives

$$\text{curl}(u_{\text{aff}} \times X) = u_{\text{aff}} \text{div } X - X \text{div } u_{\text{aff}} + (X \cdot \nabla)u_{\text{aff}} - (u_{\text{aff}} \cdot \nabla)X = 3u_{\text{aff}}.$$

Thus $u = u_{\text{aff}}$ wherever $\chi = 1$, and $\text{div } u = 0$ everywhere. Set

$$f = \partial_t u + (u \cdot \nabla)u - \nu \Delta u + \nabla p. \quad (14)$$

All fields in this formula are smooth with compact spatial support. Since B and all its derivatives vanish outside a compact time subinterval of $(0, T)$, the velocity, pressure, and force have the required time support. The force is identically zero wherever $\chi = 1$ by Theorem 3.1, and wherever χ is identically zero. It is therefore supported in the fixed transition region outside the marked trajectories.

The smooth velocity is uniformly bounded and compactly supported in space-time, giving a uniform finite-energy bound directly. By construction (7) holds exactly. Each proposed marked curve solves its particle ODE in the region where $u = u_{\text{aff}}$; uniqueness for the smooth ODE identifies it with the actual global trajectory. $\square$

Corollary 6.2 (Energy bound and localization scope). *Choose $0 \leq \chi \leq 1$, supported in $|X| \leq R_1$. Set $M_B = \sup_t \|B(t)\|_2$ and $M_\chi = \|\nabla \chi\|_\infty$. The constructed solution satisfies*

$$\sup_t \frac{1}{2} \int_{\mathbb{R}^3} |u|^2 dX \leq \frac{2\pi R_1^3}{3} M_B^2 \left(R_1 + \frac{M_\chi R_1^2}{3} \right)^2.$$

It also satisfies the exact energy identity

$$\frac{d}{dt} \frac{1}{2} \int |u|^2 dX + \nu \int |\nabla u|^2 dX = \int f \cdot u dX.$$

The cubic J is conserved along the marked trajectories. It need not be conserved along trajectories in the cutoff transition region.

Proof. The product rule gives $u = \chi BX + \nabla \chi \times \mathcal{A}$ and $|\mathcal{A}| \leq \|B\|_2 |X|^2/3$. Bound $|u|$ on its support and multiply its squared bound by half the volume of the ball. Smooth rates on the compact time interval make M_B finite; constancy near the endpoints gives the stated compact time support. Dotting (7) with u and integrating gives the energy identity: compact support eliminates boundary terms, while incompressibility eliminates the advection and pressure contributions. The marked paths stay in the affine region, where Theorem 3.1 applies. To see why the conclusion is local, take $\chi(X) = h(|X|^2)$. Euler's identity $X \cdot \nabla J = 3J$ and $(BX) \cdot \nabla J = 0$ give

$$u \cdot \nabla J = -2h'(|X|^2)(X \cdot BX)J,$$

which is generally nonzero where the cutoff varies. $\square$

Since the constructed velocity vanishes at both temporal endpoints, integrating the energy identity yields $\int_0^T \int f \cdot u = \nu \int_0^T \int |\nabla u|^2$. The global force supplies the dissipated energy, although it vanishes near the marked trajectories in the irrotational-core construction.

6.1 Vorticity, energy input, and viscosity dependence

Proposition 6.3 (The localized field has an irrotational core and a vortical collar). *For the symmetric matrix $B(t)$ of Theorem 3.1, a radial cutoff $\chi(X) = h(r^2)$, and $r^2 = X \cdot X$, formula (13) becomes*

$$u = \left(h + \frac{2}{3} r^2 h' \right) BX - \frac{2}{3} h' (X \cdot BX) X, \quad (15)$$

$$\text{curl } u = \frac{14h' + 4r^2 h''}{3} X \times BX. \quad (16)$$

All functions of h are evaluated at r^2 . More generally, for every smooth spatial cutoff,

$$u \cdot \nabla J = -J(BX) \cdot \nabla \chi. \quad (17)$$

Every nonzero smooth compactly supported divergence-free velocity has nonzero curl somewhere. Thus the nontrivial localized golden field is not globally irrotational, even without the swirl variant below.

Proof. Use $u = \chi BX + \nabla \chi \times (BX \times X)/3$ and the vector triple product with $\nabla \chi = 2h'X$ to obtain (15). Symmetry gives $\text{curl}(BX) = 0$ and $\nabla(X \cdot BX) = 2BX$. Taking the curl then gives (16) by the product rule. For a general cutoff, the same triple product gives

$$u = \left(\chi + \frac{X \cdot \nabla \chi}{3} \right) BX - \frac{(BX) \cdot \nabla \chi}{3} X.$$

Dot with ∇J , using $(BX) \cdot \nabla J = 0$ and $X \cdot \nabla J = 3J$. Finally, if a compactly supported smooth u had both zero divergence and zero curl, the vector Laplacian identity would give $\Delta u = 0$. Integration by parts would imply $\int |\nabla u|^2 = -\int u \cdot \Delta u = 0$, hence u is constant and, by compact support, zero. $\square$

Proposition 6.4 (Dissipation and necessary work). *The affine core satisfies*

$$\Delta u_{\text{aff}} = 0, \quad |\nabla u_{\text{aff}}|^2 = \|B\|_F^2 = 6\gamma^2 + 2\beta^2.$$

For every nontrivial localized motion and $\nu > 0$,

$$\int_0^T \int_{\mathbb{R}^3} f_\nu \cdot u \, dX \, dt = \nu \int_0^T \int_{\mathbb{R}^3} |\nabla u|^2 \, dX \, dt > 0. \quad (18)$$

In particular its force cannot be a spatial gradient throughout the motion.

Proof. The gradient is B and the Laplacian of a linear field is zero. Using $K^2 = I_2$, $\text{tr } K = 0$ and the axial entry -2γ gives the Frobenius norm. Integrate Corollary 6.2 in time; the endpoint kinetic energies vanish. If the last integral were zero, continuity and compact spatial support would force $u = 0$ everywhere, a contradiction. A gradient force has $\int \nabla q \cdot u = -\int q \, \text{div } u = 0$ at each time, contradicting (18). $\square$

A vanishing viscous momentum term does not mean zero strain or zero local dissipation. In the Laplacian form of the energy equation, $u \cdot \Delta u = \Delta(|u|^2/2) - |\nabla u|^2$; the two terms on the right cancel in the affine core. In the Cauchy-stress form, with $D = (\nabla u + \nabla u^T)/2$, the dissipation is $2\nu D : D$ and the energy flux is correspondingly different. The global integrals agree for the compactly supported incompressible field. Pressure work and transport flux allow energy exchange with the core even where its body force vanishes.

Proposition 6.5 (One designed force for each viscosity). *For the velocity and pressure in Theorem 6.1,*

$$f_{\nu_2} - f_{\nu_1} = -(\nu_2 - \nu_1)\Delta u.$$

A nonzero smooth compactly supported velocity cannot solve the momentum equation with the same force at two distinct viscosities, even if the pressures are allowed to differ.

Proof. Subtract (14) to obtain the displayed identity. If the same force worked with pressures p_1, p_2 , subtraction would instead give $-(\nu_2 - \nu_1)\Delta u + \nabla(p_2 - p_1) = 0$. Dot with u and integrate in space. Compact support and incompressibility eliminate the pressure term and imply $(\nu_2 - \nu_1) \int |\nabla u|^2 = 0$. Thus $u = 0$. $\square$

The existence quantifiers are therefore: a prescribed path determines u, p , and for each viscosity one computes its corresponding f_ν .

Proposition 6.6 (Cost of repeated contraction with a fixed schedule). *Fix an admissible $0 \leq \sigma \leq 1$ increasing from zero to one, a compact marked set in $z = 0$, and the paths $c_n = \lambda^{n\sigma}$, $\eta_n = 0$ for positive integers n . A single spatial cutoff can serve every n . For that choice, $B_n = nB_1$, $u_n = nu_1$, and both kinetic energy at each time and total viscous work over $[0, T]$ scale by n^2 .*

Proof. Since $0 < \lambda < 1$, all planar marked trajectories remain in the same ball as the initial set; a fixed larger ball and cutoff suffice. Differentiating $\log c_n = n\sigma \log \lambda$ gives $B_n = nB_1$. The curl localization is linear in B , hence $u_n = nu_1$. Kinetic energy and $\int |\nabla u_n|^2$ are quadratic; apply (18) for the total work. $\square$

This is an exact cost statement for these schedules and cutoffs, not an optimal lower bound over all realizations. It supplies no uniform energy bound as $n \rightarrow \infty$ at fixed duration. The common-support argument here uses planarity; points with nonzero initial z undergo axial expansion.

Proposition 6.7 (Rigid placements of the fluid realization). *Let $\alpha \in \mathbb{R}^3$ be fixed and $R \in SO(3)$. If (u, p, f) is the preceding solution, then, with $Y = R^T(X - \alpha)$,*

$$u^*(X, t) = Ru(Y, t), \quad p^*(X, t) = p(Y, t), \quad f^*(X, t) = Rf(Y, t)$$

solve the same equation and transport the placed marked set $\alpha + R\mathcal{K}$ by $X(t) = \alpha + R\Psi_{t,0}R^T(X_0 - \alpha)$.

Proof. The fixed orthogonal transformation preserves divergence, Laplacian, gradients, volume measure and vector inner products. The chain rule therefore transforms every momentum term by R . Substituting the stated particle path into its ODE gives the trajectory formula. Any ordered orthonormal planar frame in $\mathbb{R}^3$ extends to such an R by adjoining its oriented unit normal. $\square$

Explicit cutoffs. Nothing depends on an unspecified discontinuous cutoff. Put $b(s) = e^{-1/s}$ for $s > 0$ and $b(s) = 0$ for $s \leq 0$, and $H_0(s) = b(s)/(b(s) + b(1 - s))$. Then H_0 is smooth, zero for $s \leq 0$ and one for $s \geq 1$. For $0 < t_0 < t_1 < T$, $\sigma(t) = H_0((t - t_0)/(t_1 - t_0))$ gives an admissible schedule. Choose $R_0 < R_1$ with $\mathcal{T}$ strictly inside the ball of radius R_0 ; then

$$\chi(X) = 1 - H_0\left(\frac{|X|^2 - R_0^2}{R_1^2 - R_0^2}\right)$$

is a suitable spatial cutoff.

Remark 6.8 (Scope of the existence result). The fields and force are designed together. This proves a geometric realization theorem for smooth forced flows. It does not claim the same motion for an independently prescribed force. The path has finite duration, bounded deformation, and positive c ; no singular endpoint is included. The global fields are explicitly smooth for all time.

7 A vortical similarity variant

The preceding golden family is a strain flow. A separate variant adds actual rotation while preserving the marked planar shape.

Proposition 7.1 (Swirl, strain, and its exact force). *For smooth rates γ, ω , let*

$$B_s = \begin{pmatrix} \gamma & -\omega & 0 \\ \omega & \gamma & 0 \\ 0 & 0 & -2\gamma \end{pmatrix}, \quad u_s = B_s X, \quad p_s = -\frac{1}{2} X^T \text{sym}(\dot{B}_s + B_s^2) X.$$

Then $\text{div } u_s = 0$, $\text{curl } u_s = 2\omega e_z$, and

$$\partial_t u_s + (u_s \cdot \nabla) u_s - \nu \Delta u_s + \nabla p_s = (\dot{\omega} + 2\gamma\omega)(-y, x, 0).$$

If $\gamma = \dot{c}/c$, the particle map is $(w, z) \mapsto (cR_{\dot{\theta}}w, c^{-2}z)$, with $\dot{\theta} = \omega$ and identity initial conditions. It preserves the OMEGA length ratio. The core force vanishes precisely when $(c^2\omega)' = 0$.

Proof. The symmetric part of $\dot{B}_s + B_s^2$ has diagonal entries $\dot{\gamma} + \gamma^2 - \omega^2$ twice and $-2\dot{\gamma} + 4\gamma^2$. The remaining skew part is $(\dot{\omega} + 2\gamma\omega)$ times the planar rotation generator. This proves the equation and direct differentiation gives the curl and particle map. Rotation and uniform planar scaling preserve all marked length ratios. $\square$

For a start-from-rest realization with nonzero later swirl, choose smooth compactly supported rates and retain the displayed core force. The curl localization still gives smooth compact forcing and finite energy, but the force need not be confined to the collar. If the core force is zero and ω initially vanishes, $(c^2\omega)' = 0$ prevents it from spontaneously becoming nonzero. Generic rotations do not preserve the golden form $\mathcal{N}$; this proposition is the shape-preserving vortical branch.

8 Transport to the scalar, integer, and spiral branches

8.1 Exact scale and residual coordinates

For positive scales s_i , define on $\mathbb{R}^m$

$$E_s(Y) = (\lfloor Y_i/s_i \rfloor, Y_i - s_i \lfloor Y_i/s_i \rfloor)_i, \quad D_s(d, \rho) = (s_i d_i + \rho_i)_i.$$

Proposition 8.1 (Exact coordinate transport). *The maps E_s and D_s are inverse bijections between $\mathbb{R}^m$ and $\prod_i (\mathbb{Z} \times [0, s_i))$. For any self-map F of a real state space $X \subseteq \mathbb{R}^m$, put $\hat{F} = E_s F D_s$ on $E_s(X)$. Then*

$$\hat{F}^{\circ n} = E_s F^{\circ n} D_s, \quad D_s \hat{F}^{\circ n} E_s = F^{\circ n} \quad (n \geq 0).$$

Proof. The floor inequality gives the remainder bound and $D_s E_s(Y) = Y$. Conversely, $0 \leq \rho_i < s_i$ implies $\lfloor (s_i d_i + \rho_i)/s_i \rfloor = d_i$, so $E_s D_s$ is the identity. Cancel adjacent $D_s E_s$ factors, or induct on n . $\square$

Apply this proposition to a fixed material map Ψ_{t_1, t_0} . For a sequence of differing maps, the same cancellation gives the corresponding ordered composition. This is exact real or symbolic encoding; no finite representation of an arbitrary real residual or an entire fluid field is implied.

8.2 Ordered integer ledgers and geometric reconstruction

Theorem 8.2 (Admissible ledger reconstruction). *Let $A \in \mathbb{Z} \setminus \{0\}$, $B \in \mathbb{Z}_{>0}$, and $d_0 \in \mathbb{Z}$. Define*

$$d_{j+1} = \left\lfloor \frac{A d_j}{B} \right\rfloor, \quad \rho_j = A d_j - B d_{j+1} \quad (0 \leq j < n).$$

Then $0 \leq \rho_j < B$ and the terminal state d_n together with the forward-ordered list $(\rho_0, \dots, \rho_{n-1})$ reconstructs d_0 uniquely by exact backward division. More generally, a proposed record is the encoding of a unique integer seed if and only if each backward step has $0 \leq \rho_j < B$ and $A \mid (Bd_{j+1} + \rho_j)$. Also

$$A^n d_0 = B^n d_n + \sum_{j=0}^{n-1} B^j A^{n-1-j} \rho_j.$$

Proof. The floor inequality yields the bounds and $Ad_j = Bd_{j+1} + \rho_j$. As $A \neq 0$, the only possible predecessor is $(Bd_{j+1} + \rho_j)/A$, which is integral precisely under the divisibility condition. Conversely, for that integer predecessor, dividing the identity by the positive B shows that its forward floor is d_{j+1} and its remainder is ρ_j . Induction backward proves existence and uniqueness of a valid record; any actual encoding satisfies all the tests. Substitution of successive one-step identities proves the weighted formula by induction. $\square$

This includes signed seeds. It does not assert extinction of negative seeds. An unchecked integer-division routine must not be substituted for the exact divisibility tests when accepting a proposed record.

Fix a center α , an ordered isometric planar frame E , and a radius grid scale $h_a > 0$. If the radius parameter is $a = h_a d_0 > 0$, the ledger recovers the entire specified marked curve

$$\alpha + E\Gamma_\Omega(h_a \text{ recover}_{A,B}(d_n; \rho_0, \dots, \rho_{n-1})) = \alpha + E\Gamma_\Omega(h_a d_0).$$

The proof is substitution of Theorem 8.2. The curve specification, center, frame and h_a are retained inputs, not data inferred from the ledger. This is precisely the geometric role of the supplied Lean theorem `Omega.curve_reconstruction`.

8.3 The certified scalar grid interface

Set $Q = 339949771344778$, $S = 10^{15}$, $q = Q/S$, and $F(d) = \lfloor Qd/S \rfloor$. For a dimensionless observation with $L(w) = hd$, $h > 0$, $d \in \mathbb{Z}_{\geq 0}$, a relative similarity step $c = \lambda$ gives $L(w') = \lambda hd$. Here h is a dimensionless observation scale, separate from the physical radius grid h_a . A quantized register therefore updates by

$$\mathcal{Q}_h(hd) = h \lfloor \lambda d \rfloor.$$

The certified equality

$$\lfloor \lambda d \rfloor = F(d) \quad (0 \leq d < 42189065) \tag{19}$$

is proved by the exact certificate in Appendix A. The supplied Lean and Julia versions cover $0 \leq d \leq 32767$; the larger interval is a separate certificate and is not attributed to those versions. Because $0 < Q < S$, $F(d) < d$ for positive d , so each stated initial interval is forward invariant. Iterating the quantized register gives $\mathcal{Q}_h^{on}(hd) = hF^{on}(d)$ there. Theorem 8.2 with $(A, B) = (Q, S)$ reconstructs each integer register state exactly.

The real and rational remainders are different even where their floors agree:

$$\lambda d - F(d) = \frac{Qd - SF(d)}{S} + (\lambda - q)d. \tag{20}$$

For $d > 0$ the correction is strictly positive. The stored integer remainder is the residual of the rational map.

Per-step quantization is part of this register's definition. It is not the uninterrupted material trajectory sampled only at its final time. For example,

$$F(F(26)) = 2, \quad \lfloor q^2 26 \rfloor = 3, \quad \lfloor \lambda^2 26 \rfloor = 3.$$

Likewise $F(1) = 0$ although a positive material scale remains positive under any finite similarity step. Discrete extinction is thus not physical collapse of the curve. The equation supplies a scalar observation; the quantized algorithm and its ledger are additional specified operations.

8.4 Sacks and golden-parameter observations

Keep the supplied definitions

$$g(k) = k^2 - k - 1, \quad Z(t) = \sqrt{t} e^{2\pi i \sqrt{t}}, \quad \mathcal{W}(z) = e^{-2\pi i |z|} z,$$

$$\mathcal{R}_c(z) = c e^{2\pi i (c-1)|z|} z, \quad \mathcal{K}_c(k) = \frac{1}{2} + \sqrt{c^2 g(k) + 5/4}, \quad k \geq \varphi.$$

For $c > 0$, $t \geq 0$ and $k \geq \varphi$, these definitions give

$$\mathcal{W}\mathcal{R}_c = c\mathcal{W}, \quad \mathcal{R}_c Z(t) = Z(c^2 t), \quad g\mathcal{K}_c = c^2 g.$$

Indeed, $|\mathcal{R}_c(z)| = c|z|$, so its phase cancels under $\mathcal{W}$, giving $\mathcal{W}\mathcal{R}_c(z) = c\mathcal{W}(z)$. Substitution of $Z(t)$ gives $\mathcal{R}_c Z(t) = c\sqrt{t} e^{2\pi i c\sqrt{t}} = Z(c^2 t)$. Finally $g(k) = (k - 1/2)^2 - 5/4$ gives $g(\mathcal{K}_c(k)) = c^2 g(k)$ directly; the positive square-root branch places $\mathcal{K}_c(k)$ back in $[\varphi, \infty)$. The inverse of $\mathcal{W}$ is $z \mapsto e^{2\pi i |z|} z$, so the first identity is an exact conjugacy.

They share the planar scale c with our material-similarity subgroup. For example, its motion on a fixed nonnegative ray in the unwound plane becomes $\mathcal{R}_c$ when displayed in Sacks coordinates. These are exact representation maps. Ulam integer tracks, prime-divisibility classes, and the Fibonacci/Lucas polynomial discriminants remain the supplied arithmetic statements; continuous fluid transport does not preserve integer labels.

8.5 The physical Navier–Stokes scaling law

An independent, standard equation symmetry must be distinguished from particle deformation. If (u, p, f) solves (7), define for $c > 0$

$$u_c(X, t) = c^{-1} u(X/c, t/c^2), \quad p_c(X, t) = c^{-2} p(X/c, t/c^2), \quad f_c(X, t) = c^{-3} f(X/c, t/c^2).$$

The transformed fields solve the same equation with the same viscosity. Indeed each momentum term acquires c^{-3} and the divergence acquires c^{-2} . This maps one solution to another and rescales time. It is not the volume-preserving motion of one parcel in a fixed solution.

Likewise, using the nonlinear Sacks map as a spatial coordinate change requires transporting the differential operators and metric. For example, on $r > 0$ use the chart map that sends the parameters (r, θ, z) to Euclidean cylindrical coordinates $(r, \theta - 2\pi r, z)$. Its pullback metric is

$$ds^2 = dr^2 + r^2(d\theta - 2\pi dr)^2 + dz^2.$$

The connection, divergence, and Laplacian must be those of this metric in the transformed equations. The global homeomorphism used for the original spiral theorem alone is insufficient to justify a smooth PDE change of coordinates through the axis. The Sacks display uses dimensionless coordinates. For a physical radial coordinate with length units, first choose a reference length ℓ_0 and replace the twist $2\pi r$ by $2\pi r/\ell_0$, and $2\pi dr$ in the metric by $(2\pi/\ell_0)dr$.

9 Golden norm, fluctuation stress, and exact cubature

Proposition 9.1 (An exact quadratic observation of a fluctuation stress). *Let $v = \bar{v} + w$ be a planar random or phase-dependent vector with finite second moment, $\langle w \rangle = 0$, and let $R = \langle ww^T \rangle$. Then*

$$\boxed{\langle \mathcal{N}(v) \rangle - \mathcal{N}(\bar{v}) = \text{tr}(HR)}. \quad (21)$$

Moreover,

$$|\text{tr}(HR)| \leq \frac{\sqrt{5}}{2} \text{tr} R.$$

Under the normalized Fibonacci transformation $v \mapsto A_\Omega v$, the covariance becomes $A_\Omega R A_\Omega^T$ and its golden observation becomes $-\tau_\Omega^2 \text{tr}(HR)$.

Proof. Expand $(\bar{v} + w)^T H(\bar{v} + w)$ and average. The cross terms vanish and $\langle w^T H w \rangle = \text{tr}(HR)$. Since $H^2 = (5/4)I_2$, its eigenvalues are $\pm\sqrt{5}/2$. For the positive semidefinite covariance R , the claimed bound follows by diagonalizing H . Finally, $A_\Omega^T H A_\Omega = -\tau_\Omega^2 H$ proves the transformation law. $\square$

For a three-dimensional velocity fluctuation, this identity applies to its two planar components and the corresponding covariance block. It is an exact algebraic observation of Reynolds stress. For example, $R = rI_2$ with $r \geq 0$ gives $\text{tr}(HR) = 0$ while $\text{tr} R = 2r$ can be arbitrarily large. Thus the displayed upper bound is not a coercive energy estimate and the scalar does not reconstruct the stress. Realizing a prescribed covariance by a velocity field subject to additional differential or boundary conditions is a separate existence problem.

Proposition 9.2 (Cubature of the affine fluid observables). *Suppose nodes $\omega_j \in S^2$ and weights w_j integrate spherical polynomials of degree at most two against surface area measure, so $\sum_j w_j = 4\pi$. For an affine core $u(X) = BX$, at radius r ,*

$$\sum_j w_j |u(r\omega_j)|^2 = \frac{4\pi r^2}{3} \text{tr}(B^T B), \quad (22)$$

$$\sum_j w_j u(r\omega_j) u(r\omega_j)^T = \frac{4\pi r^2}{3} B B^T. \quad (23)$$

Pressure from (8) is also integrated exactly. The kinetic energy of the affine core in a ball of radius R_0 is

$$\frac{1}{2} \int_{|X| \leq R_0} |BX|^2 dX = \frac{2\pi R_0^5}{15} \text{tr}(B^T B).$$

Proof. Rotational symmetry and $\sum_i \omega_i^2 = 1$ give $\int_{S^2} \omega \omega^T d\sigma = (4\pi/3)I_3$. Each integrand is a polynomial of degree two in ω , so the assumed cubature is exact. Integrating r^4 from 0 to R_0 proves the last formula. $\square$

This supplies a concrete use of the spherical-sampling branch: exact angular energy, pressure, and stress measurements for the affine core. It does not assert exact integration of the cutoff transition region by the same fixed degree rule. Equal weights on the 12 normalized icosahedral vertices already suffice for the moments used here; the companion script checks them exactly. Four regular tetrahedral nodes also suffice: take $(1, 1, 1), (1, -1, -1), (-1, 1, -1), (-1, -1, 1)$ divided by $\sqrt{3}$, each of weight π . Their sum is zero and their outer products sum to $(4/3)I_3$, proving exactness for constants, linear functions and quadratics. Thus the golden or icosahedral choice is not necessary for these degree-two observables. The previously reported higher-degree cubature certificates are not needed for this proposition and were not rerun for this extension.

10 Assembled geometric transport theorem

Theorem 10.1 (OMEGA-compatible geometric fluid extension). *The supplied geometry, golden algebra, contraction, and frozen exact encoder extend to the family (3) with the following proved connections:*

1. *Its similarity subgroup transports a material OMEGA construction and preserves $(G + C)/a = \Omega_0$; its scalar observation is the OMEGA contraction.*
2. *Its golden anisotropic subgroup realizes M^2 and A_Ω^2 , with the observation and norm laws in Corollary 5.1.*
3. *The full family preserves three-dimensional volume and the cubic material invariant $z\mathcal{N}(w)$.*
4. *Every smooth finite path in (3), with positive scale, identity initial map and schedules constant on neighborhoods of both endpoints, has the finite-energy, smooth compact-force realization of Theorem 6.1.*
5. *Exact encoding transports these finite-dimensional material states without altering SRSD. The certified floor ledger applies to the specified quantized scalar observation.*
6. *The same scale parameter has the existing Sacks/golden-parameter representations; golden quadratic stress and affine cubature identities hold as stated in Propositions 9.1 and 9.2.*

Proof. Each item is the indicated theorem or proposition. Composition is only on the stated domains. Material states, velocity fields, integer samples, covariances, and geometric display coordinates remain different types. $\square$

Existing branch	Added exact connection	Intended site location
Marked geometry and Ω_0	Material planar similarity; constant normalized length	geometry, omega0
Contraction	$\tilde{V}\Phi_{\lambda,0} = T\tilde{V}$	dynamics
Golden norm and matrix	Volume-preserving lift; invariant $z\mathcal{N}$; A_Ω^2 step	network plus supplied extension
Rational floor ledger	Explicit scalar grid observation with per-step quantization	dynamics
Frozen SRSD	Exact encoding of the specified material state	srsd
Sacks and norm-polynomial tracks	Coordinate representations using the same c	Supplied golden-norm dossier
Icosahedral cubature	Exact degree-two affine energy and stress measurements	cubature
Verification contract	Named hypotheses and exact symbolic identities	contract

10.1 Scope of the fluid conclusion

Theorem 6.1 constructs a regular, finite-energy solution together with its smooth force. Its proof uses a compact trajectory set, smooth bounded rates on a finite interval, and positive scale. No singular endpoint is included. The standard equation and finite-energy setting are those in [6]; no external finite-time singularity manuscript is a premise of this paper.

The integer remainder, the fluctuation covariance, and the PDE force residual are different mathematical objects. The first reconstructs an integer predecessor; the second records a quadratic mean effect; the third enforces the momentum balance for a prescribed smooth velocity and pressure. Their exact formulas provide connections without identifying their domains or acceptance conditions.

10.2 What is forced to remain constant

For the shape-preserving branch, the geometric identity (1) and the proved fluid transport force the readout to remain Ω_0 . For the golden family, the conserved material scalar is $J = z\mathcal{N}(w)$. These are conditional invariants of explicitly constructed flows. Navier–Stokes has not independently selected $\varphi + \pi$: its value enters through the initial geometric construction and the chosen rates. The result is an exact realization and preservation theorem with those choices visible.

Proposition 10.2 (The cubic invariant neither determines shape nor survives arbitrary strain perturbations). *On the marked plane $z = 0$, the value of J is identically zero. For $D_0 = \text{diag}(1, -1, 0)$, replacing B by $B + \epsilon D_0$ gives*

$$\frac{d}{dt}J(X(t)) = 2\epsilon z(x^2 + y^2).$$

The perturbed affine velocity remains incompressible and symmetric in its spatial gradient. For $\epsilon \neq 0$ it generally fails to preserve J .

Proof. The first assertion follows from the factor z . The new contribution to the derivative is $\epsilon z[x(2x + y) - y(x - 2y)] = 2\epsilon z(x^2 + y^2)$. Both B and D_0 are symmetric and traceless. $\square$

Preservation of the marked shape therefore follows from the similarity map, not from the value of J . The invariant is a special algebraic first integral of the chosen family; no robustness or attraction theorem for nearby general fluid motions is claimed.

11 Integration of the supplied Lean and Julia packages

Two supplied artifacts give a concrete foundation for the geometric and arithmetic interfaces: the Word transcription [1] and the Julia companion [2]. They can be integrated as source and verification layers, with the fluid arguments of this paper added as a separate mathematical layer.

11.1 Source identity and mathematical interfaces

The Word document contains 29 file blocks totaling 98,418 UTF-8 bytes. Recovering the source paragraphs and their line breaks gives byte-for-byte equality with all 29 files in the Julia package’s `reference/Lean/` directory. The Julia archive contains 68 files. Its 67 listed SHA-256 entries match, as do the 28 entries in the nested Lean manifest. A manifest cannot authenticate its own provenance; these checks establish internal consistency and equality between the supplied copies.

The 147 entries in `THEOREMS.toml` match the 147 named source declarations, including their stated hypotheses. The following is the integration actually used here. The names identify supplied source declarations; they are not claims that the fluid theorems have been newly compiled in Lean.

Source interface	Role in this paper
<code>Geometry.lean: G_eq, C_eq, W_eq, reference_not_outline</code>	Precise marked pieces and selected measurement; restated and proved in Proposition 1.1.
<code>Placement.lean: provenance_all_placements, placed_homothety, readout_scale</code>	Isometric placements and shape-preserving similarity; connected to the fluid through Corollary 4.1 and Proposition 6.7.
<code>Affine.lean: weighted_mean, observation_commutates</code>	The declared weighted readout and scalar observation of a homothety; used on the planar similarity branch.
<code>Floor.lean: certified_floor_equality;</code>	The certified 15-bit register update and the distinction between rational and real residuals in (20).
<code>Interface.lean: grid_trajectory, real_residual</code>	
<code>Ledger.lean: roundtrip, admissibility_iff;</code>	Ordered exact division, unique admissible seeds, and reconstruction of the specified curve when scale, center and frame are retained.
<code>Placement.lean: curve_reconstruction</code>	
<code>Julia src/Dynamics.jl, src/Geometry.jl, src/ExactArithmetic.jl</code>	Executable integer and rational interfaces, including checked ledger decoding; analytic real-valued statements remain in the written proofs.

11.2 What is inherited and what is added

The supplied Lean project pins Lean 4.19.0 and includes a successful archived build log and an axiom report for 147 declarations. Its recorded mathlib revision is `c44e0c8ee63ca166450922a373c7409c5d26b00b`. The supplied Julia project records Julia 1.11.6 as the tested version and includes archived runs at optimization levels 0 and 3. Its project compatibility condition is Julia 1.11; this is not a patch-version pin. Its proof guide explicitly distinguishes written universal arguments, executable exact certificates, and numerical illustrations. Reading a proof string or a log is not a new kernel check.

The new objects in this paper are the matrix K , the volume-preserving family Φ , its material invariant and relative-time flow, the affine fluid field, and its localized finite-energy realization. These statements are proved here and checked algebraically where applicable by the revised SymPy companion. Neither attachment supplies a Lean formalization or a Julia implementation of these new PDE results. Integration therefore extends the mathematical dependency structure without relabeling old evidence as a new fluid certificate.

A single scalar observation is not injective on the fluid state. Curve recovery from an integer ledger works because the curve is a specified one-parameter family and its placement data are retained. Similarly, the exact scalar interface is compatible with the frozen SRSD convention; it does not modify that foundation. The supplied packages do not contain a new formalization of the Sacks or golden matrix extension, so those identities are justified by this paper and the separate supplied golden-norm dossier [3].

12 Verification and reproducibility

Fresh computations used Python 3.12.14 and SymPy 1.14.0. All acceptance checks use explicit exceptions and remain active with Python optimization enabled. The revised `verify_omega_ns.py` checks 62 exact algebraic and regression conditions, including relative initial-time normalization, the constant-schedule counterexample to the old formula, dimensionless observations, the transition region’s cubic defect, the general covariance law, and the full affine stress moment. Version 3.0 adds 17 checks for the radial velocity and collar curl, core energy density, the general cutoff defect, strain perturbations, isotropic-stress

blindness, tetrahedral moments, and the arbitrary-real-parameter family. It also runs the independent exact arithmetic audit for the first floor mismatch and the previously stated byte and tick identities.

```
python verify_omega_ns.py
python -O verify_omega_ns.py
```

Both modes completed with identical output. These runs check algebra and exact arithmetic; the smooth cutoff, compactness, ODE uniqueness and energy arguments are the analytic proofs given in this paper.

The companion `audit_omega_sources.py` accepts the supplied Word file and ZIP as its two positional arguments. It requires the complete fixed-release file and DOCX-block inventories, rejects duplicate blocks and manifest entries, and binds each mapped theorem to its own name, module, source line, exact declaration and recorded section variables. Its declaration extractor is restricted to the supplied source syntax; it is not a Lean parser or kernel. It checks complete ordered table coverage and unique keys before replay. Actual Julia certificate expressions, including moment comprehensions, height dependencies and indexed Gram entries, are interpreted through an allowlisted exact-arithmetic syntax. Unknown constructs are rejected. The polynomial prelude and arithmetic dependency are restricted to the inspected release. Recorded normal forms must agree with the fresh reductions. These checks reproduce the following certificates in Python. The checks against the attachments require Python 3.11 or later for the standard library TOML parser and SymPy 1.14.0.

Fresh attachment check	Result
DOCX source blocks equal the nested Lean files	29 / 29
Outer Julia and nested Lean hash entries	67 / 67; 28 / 28
Named theorem declarations matched to the mapping	147 / 147
Archived axiom-report coverage and allowed dependencies	147 reports
Archived O0/O3 certificate files compared byte-for-byte	6 / 6
Independent integer-state census and reverse reconstruction	32,768 states
Forward steps, including retained zero tails	753,664
Maximum extinction depth; sum of depths	10; 300,871
Exact rational enclosure rows reproduced	10 / 10
Real-floor comparisons using enclosing rationals	32,768
Polynomial expressions independently reduced with SymPy	97 / 97
Precision sweep rows and gap-block rows reproduced	19; 128

The axiom-report check finds only `Classical.choice`, `Quot.sound` and `propext` in the archived report. This is a check of that report, not a fresh Lean build. No Lean or Julia executable was available in the present environment; neither runtime was rerun. The package’s additional generic-ledger and numerical-illustration populations remain archived evidence, except for the specific arithmetic and algebra independently reproduced above.

The canonical replay includes 300,871 positive transitions and 452,793 zero transitions; its minimum upward gap numerator is 34,535,026,732. Optimization-level equality is not counted as a second independent mathematical proof. The 97 polynomial checks use $\varphi^2 = \varphi + 1$ and $c_\delta^2 + s_\delta^2 = 1$ as established relations; reducing those relations to zero is a consistency check, not an independent derivation of the golden root or trigonometry.

Adversarial regression. The accompanying `test_omega_auditor.py` creates disposable mutated copies and refreshes their internal manifests after semantic edits. It runs one valid case and 18 invalid cases, each under ordinary Python and `python -O`: 38 subprocess checks, comprising two acceptances and 36 required rejections. The five previously demonstrated false passes are rejected: absent DOCX source

blocks, a changed moment expression, empty evidence tables, a theorem paired with another theorem’s statement, and a false recorded polynomial normal form. Additional cases isolate missing precision and gap coverage, duplicate source blocks and rows, changed moments/Gram entries/dependencies, incomplete manifests, incorrect section context, and unsupported syntax. These are observed regressions, not a proof that arbitrary adversarial archives are safe or correct.

The supplied artifacts were identified by the following SHA-256 digests:

```
LEAN_Proof_Complete_Final(1).docx
275da4ea14130d21aa1b797b643ec78192e195780bfd8352bac699a75b73542e

Julia_OMEGA_NAUGHT_Julia_Proof_v0.1(1).zip
ef8f1eb75b83f1e5763d280374eb8b73d9ab00b8dcb76b1824aee771869cd27a
```

The website links in the integration table identify relevant archive locations. The live site could not be retrieved during this revision; source claims here are grounded in the supplied files, not in a new live-site audit.

Reproduction supplement. This PDF embeds the two unchanged source archives, the revised auditor and symbolic verifier, the mutation-test harness, the editable L^AT_EX source, and `OMEGA_V3_Verification_Record.txt`. The record contains exact commands, software versions, observed outputs and SHA-256 identities. Use a PDF viewer’s attachments panel to save the files, or Poppler’s `pdftdetach -saveall` in an empty directory. Python 3.11+ and SymPy 1.14.0 suffice for the checks. No public repository identifier or DOI is assigned to this revision; the embedded files are the supplied computational supplement. The manuscript source builds with two `pdflatex` passes; the supplement’s build script adds the attachments with `pypdf`.

Version record. Version 3.0 retains the relative-time, geometry, ledger and floor-boundary corrections of Version 2.0. It repairs the auditor’s identified false passes, adds reproducible negative tests, strengthens endpoint and viscosity wording, proves the collar-vorticity and energy limitations, and establishes the general parameter and perturbation results. The original Lean and Julia archives remain unchanged. All fresh acceptance counts in this section refer to this revision’s executions.

Preparation. This manuscript and its computational checks were prepared with AI assistance. Mathematical acceptance rests on the displayed arguments, specified hypotheses and reproducible calculations. No external peer review or new proof-assistant verification is represented by this preparation record.

A An exact certificate for the scalar floor boundary

This appendix records the separate integer-arithmetic certificate supporting (19); it extends beyond the 15-bit domain of the supplied Lean and Julia versions. Let

$$M_0 = 10^{30}, \quad L = 339949771344778929576702809321, \quad U = L + 1, \quad N_* = 42189065.$$

Theorem A.1 (First nonnegative floor disagreement). *For every integer $0 \leq d < N_*$, $\lfloor \lambda d \rfloor = \lfloor Qd/S \rfloor$. At $d = N_*$,*

$$\lfloor \lambda N_* \rfloor = 14342163, \quad \lfloor QN_*/S \rfloor = 14342162,$$

so N_* is the first disagreement on the nonnegative integers.

Proof. All bounds and sums below are calculated with integers and rational numbers. Let $R = \lfloor 10^{80} \sqrt{5} \rfloor$, computed as `isqrt(5 · 10160)`. Squaring certifies $R/10^{80} < \sqrt{5} < (R + 1)/10^{80}$. For $b = 5, 239$, let

$$A_b = \sum_{j=0}^{63} \frac{(-1)^j}{(2j+1)b^{2j+1}}, \quad B_b = A_b + \frac{1}{129b^{129}}.$$

The alternating-series remainder gives $A_b < \arctan(1/b) < B_b$. Machin's identity gives

$$16A_5 - 4B_{239} < \pi < 16B_5 - 4A_{239}.$$

For completeness, if $\theta = \arctan(1/5)$ and $\eta = \arctan(1/239)$, then $\tan(2\theta) = 5/12$, $\tan(4\theta) = 120/119$, and $\tan(4\theta - \eta) = 1$. The inequalities $x/(1+x^2) < \arctan x < x$ for $x > 0$ place $4\theta - \eta$ strictly between 0 and $4/5 < \pi/2$. Hence $4\theta - \eta = \pi/4$, establishing the identity on the required branch.

The quotient $\varphi/(\varphi + \pi)$ increases with positive φ and decreases with positive π . Combining the displayed enclosures and exact cross multiplication therefore certifies

$$\frac{Q}{S} < \frac{L}{M_0} < \lambda < \frac{U}{M_0}.$$

Define $H(n, m, a, b) = \sum_{i=0}^{n-1} \lfloor (ai + b)/m \rfloor$ for nonnegative integers n, a, b and positive m . Remove whole multiples of m from a, b , adding $(a // m)n(n-1)/2 + (b // m)n$. With $0 \leq a, b < m$, put $k, r = \text{divmod}(an + b, m)$. If $k = 0$ the remaining sum is zero. Otherwise $a > 0$, and counting integer levels under the graph gives

$$H(n, m, a, b) = \sum_{j=1}^k \left[n - \left\lceil \frac{mj - b}{a} \right\rceil \right] = \sum_{j=1}^k \left\lfloor \frac{m(k-j) + r}{a} \right\rfloor = H(k, a, m, r).$$

A possible empty top level contributes zero. The reduced modulus decreases, so this gives a terminating exact Euclidean recurrence. Evaluating it yields

$$H(N_*, M_0, U, 0) = H(N_*, S, Q, 0) = 302541195258184.$$

For every nonnegative d , the upper-floor summand is at least the rational-map summand. Their equal finite sums force equality of every summand for $0 \leq d < N_*$. Squeezing λ between Q/S and U/M_0 proves the claimed agreement. Finally direct integer divisions give

$$\left\lfloor \frac{LN_*}{M_0} \right\rfloor = \left\lfloor \frac{UN_*}{M_0} \right\rfloor = 14342163, \quad QN_* \bmod S = 999999976452570, \quad QN_* // S = 14342162.$$

These certify the first failure at N_* . □

The equality of these one-step floor maps is sometimes called restricted conjugacy in the supplied archive. Here it is used precisely as pointwise agreement of two maps on a stated interval; the exact conjugacies in this paper are the bijective coordinate transports with their domains explicitly specified. The identity $Qd = Sd' + \rho$ and its reverse remain valid beyond N_* .

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