

# OMEGA and Navier–Stokes

## Geometric Transport and Exact Invariants

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### Abstract

The supplied OMEGA geometry and golden quadratic form admit an explicit embedding in incompressible three-dimensional fluid motion. A two-parameter family of volume-preserving deformations transports the golden norm and preserves the cubic quantity  $z(x^2 + xy - y^2)$ . Its similarity subgroup transports the entire marked planar geometry while keeping  $(G + C)/a = \varphi + \pi$  constant. Its anisotropic subgroup realizes the square of the normalized Fibonacci matrix and its scalar contraction observation. Smooth time paths have explicit affine Navier–Stokes realizations. A curl localization gives finite-energy, globally smooth forced realizations, starting from rest, for any prescribed finite motion of a compact marked set. Further identities connect the construction to the frozen residual encoder, Sacks transport, fluctuation stresses, and spherical cubature.

**Status.** The general results below are proved in this document. The companion script checks their algebra symbolically. This is not a proof-assistant certificate, a claim of historical novelty, or a new finite-time blowup result. The OMEGA value is supplied by its geometric measurements; the equations below preserve or realize it. The general construction also works with other selected scale constants.

**Source access.** The home, geometry, network, constant, contract, SRSD, dynamics, cubature, and machine-manifest endpoints of <https://omega-naught.grok.me/> were attempted on 18 September 2026. Direct retrieval returned HTTP 403; web retrieval also failed. Consequently, this extension uses the supplied thread, golden-norm TeX/PDF, and local checked synthesis, rather than claiming to audit the current website. The website links in Section 10 identify intended integration points.

**Foundational convention.** SRSD remains frozen. It is used only as the external exact encoder/decoder supplied by the existing work.

**Reading guide.** Sections 2–5 prove the geometric and fluid identities; Section 6 proves the finite-energy realization; Sections 7–9 connect swirl, exact encoding, spiral coordinates, stresses, and cubature; Section 10 assembles the result and states its relation to the external blowup claim.

# 1 Definitions and the geometric question

Write

$$\varphi = \frac{1 + \sqrt{5}}{2}, \quad \varphi' = \frac{1 - \sqrt{5}}{2}, \quad \Omega_0 = \varphi + \pi, \quad \tau_\Omega = \Omega_0^{-1}, \quad \lambda = \tau_\Omega \varphi.$$

The symbol  $\tau_\Omega$  denotes a constant, not the remaining-time variable used in some fluid blowup constructions. The supplied marked planar geometry has scale  $a > 0$  and selected lengths  $G = a\varphi$ ,  $C = a\pi$ . Thus its normalized readout is

$$\mathcal{I} = \frac{G + C}{a} = \Omega_0. \quad (1)$$

These measurements and their selection rule are hypotheses of the connection. The problem here is to transport the geometry by an actual incompressible fluid velocity while preserving this readout, and to connect that transport to the existing golden algebra.

Use orthonormal spatial coordinates  $X = (x, y, z) = (w, z)$ ,  $w \in \mathbb{R}^2$ , with the initial marked construction in  $z = 0$ . A fixed rigid change of coordinates places any supplied planar OMEGA realization in this convention. Define

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \quad H = \begin{pmatrix} 1 & 1/2 \\ 1/2 & -1 \end{pmatrix}, \quad K = \frac{2M - I_2}{\sqrt{5}} = \frac{1}{\sqrt{5}} \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix}.$$

The golden quadratic form and its linear observations are

$$\mathcal{N}(w) = w^T H w = x^2 + xy - y^2, \quad \ell_+(w) = x + \varphi y, \quad \ell_-(w) = x + \varphi' y.$$

As in the supplied dossier,  $\mathcal{N}$  is indefinite; it is not kinetic energy or a positive metric norm. Direct multiplication gives

$$K^T = K, \quad \text{tr } K = 0, \quad K^2 = I_2, \quad K^T H + H K = 0, \quad \ell_+ K = \ell_+, \quad \ell_- K = -\ell_-. \quad (2)$$

Here the last two identities treat the observations as row vectors.

## 2 A volume-preserving geometric family

**Theorem 2.1** (Golden deformation and its cubic invariant). *For  $c > 0$  and  $\eta \in \mathbb{R}$ , define*

$$\Phi_{c,\eta}(w, z) = (ce^{\eta K} w, c^{-2} z). \quad (3)$$

Then

$$\Phi_{c,\eta} \circ \Phi_{d,\theta} = \Phi_{cd,\eta+\theta}, \quad \det D\Phi_{c,\eta} = 1, \quad (4)$$

$$\mathcal{N}(ce^{\eta K} w) = c^2 \mathcal{N}(w), \quad \ell_+(ce^{\eta K} w) = ce^{\eta} \ell_+(w). \quad (5)$$

In particular,

$$\boxed{J(X) = z\mathcal{N}(w) = z(x^2 + xy - y^2), \quad J \circ \Phi_{c,\eta} = J.} \quad (6)$$

*Proof.* Since all exponentials are generated by the same matrix, their products add parameters. Also  $\det e^{\eta K} = e^{\eta \text{tr } K} = 1$ , so the planar determinant is  $c^2$  and the axial factor  $c^{-2}$  compensates it. Differentiating  $e^{\eta K^T} H e^{\eta K}$  and applying (2) gives zero, hence that expression equals  $H$ . This proves the norm formula. The eigen-observation identities imply  $\ell_+ e^{\eta K} = e^{\eta} \ell_+$  and similarly  $\ell_- e^{\eta K} = e^{-\eta} \ell_-$ . Finally, multiply  $c^2 \mathcal{N}(w)$  by  $c^{-2} z$ .  $\square$

*Remark 2.2.* The invariant  $J$  concerns particle position. It is a first integral for the flow constructed below, not a claim that every Navier–Stokes velocity preserves this cubic function. On the geometry plane  $z = 0$ ,  $J = 0$ ; nonzero levels describe additional three-dimensional trajectories.

**Proposition 2.3** (Why compensation is necessary). *A differentiable incompressible particle flow has Jacobian determinant one when initialized at the identity. A uniform three-dimensional homothety  $X \mapsto cX$  can therefore be such a flow map only when  $c = 1$ . Planar scaling by  $c$  can be realized by compensating its area factor with the axial factor  $c^{-2}$  in (3).*

*Proof.* Jacobi’s formula for a flow map  $\Psi_t$  gives  $\frac{d}{dt} \det D\Psi_t = (\operatorname{div} u) \circ \Psi_t \det D\Psi_t = 0$ . The determinant of  $cI_3$  is  $c^3$ , whereas  $\det \operatorname{diag}(c, c, c^{-2}) = 1$ .  $\square$

### 3 Exact Navier–Stokes realization

We use density one and the Euclidean equations

$$\partial_t u + (u \cdot \nabla)u - \nu \Delta u + \nabla p = f, \quad \operatorname{div} u = 0, \quad \nu > 0. \quad (7)$$

Pressure is pressure divided by density. Spatial coordinates have length units; the rates below have inverse-time units.

**Theorem 3.1** (Affine realization for every positive viscosity). *Let  $c(t) > 0$  and  $\eta(t)$  be smooth, and set*

$$\gamma = \dot{c}/c, \quad \beta = \dot{\eta}, \quad B(t) = \begin{pmatrix} \gamma I_2 + \beta K & 0 \\ 0 & -2\gamma \end{pmatrix}.$$

*Then*

$$\boxed{u_{\text{aff}}(X, t) = B(t)X, \quad p_{\text{aff}}(X, t) = -\frac{1}{2}X^T(\dot{B} + B^2)X} \quad (8)$$

*solve (7) with  $f = 0$  for every  $\nu > 0$ . If  $c(0) = 1$ ,  $\eta(0) = 0$ , their particle map is  $\Phi_{c(t), \eta(t)}$ . Along every particle trajectory,  $J$  in (6) is constant.*

*Proof.*  $B$  is symmetric and traceless. Therefore

$$\operatorname{div}(BX) = \operatorname{tr} B = 0, \quad \Delta(BX) = 0, \quad \partial_t(BX) = \dot{B}X, \quad ((BX) \cdot \nabla)(BX) = B^2X.$$

Both  $\dot{B}$  and  $B^2$  are symmetric, so  $\nabla p_{\text{aff}} = -(\dot{B} + B^2)X$ . Substitution proves the entire momentum equation, including the viscous term. Differentiation of (3) gives  $\partial_t \Phi = B\Phi$  and the correct initial map. The first integral follows from Theorem 2.1; equivalently,  $(BX) \cdot \nabla J = 0$ .  $\square$

The affine velocity is irrotational because  $B$  is symmetric. Except when zero, it has infinite total kinetic energy on  $\mathbb{R}^3$ . These are exact local and whole-space algebraic solutions, not finite-energy whole-space solutions. Section 6 supplies the finite-energy forced realization. In that realization viscosity acts in the transition region even though its Laplacian vanishes in the affine core.

### 4 The OMEGA shape and scalar contraction inside the flow

**Corollary 4.1** (A material OMEGA invariant). *Use  $\eta(t) = 0$ . Any compact marked planar OMEGA construction  $\Gamma_\Omega(a_0)$  placed in  $z = 0$  is carried by the particle map to  $\Gamma_\Omega(a_0 c(t))$ . Consequently*

$$a(t) = a_0 c(t), \quad G(t) = a_0 c(t) \varphi, \quad C(t) = a_0 c(t) \pi, \quad \boxed{\frac{G(t) + C(t)}{a(t)} = \Omega_0}.$$

*Angles, marked length ratios, and the selected circular and golden features are preserved. For any linear planar observable  $L$ , the affine observation  $V(w) = 1 + L(w)$  obeys*

$$V(\Phi_{c,0}(w, 0)) = 1 + c(V(w) - 1). \quad (9)$$

*At  $c = \lambda$  this is the supplied map  $T(v) = 1 + \lambda(v - 1)$ .*

*Proof.* On  $z = 0$  the map is the Euclidean similarity  $w \mapsto cw$ . Every length scales by  $c$  and every angle remains the same. Substitute the initial measurement identities and use linearity of  $L$ .  $\square$

For example, setting  $c(t) = \lambda^{t/t_0}$  on a finite interval realizes one contraction step every duration  $t_0 > 0$ . To start and end at rest, replace  $t/t_0$  by a smooth schedule  $\sigma(t)$ , constant near the interval endpoints. The exact observation becomes  $1 + \lambda^{\sigma(t)}(V_0 - 1)$ . There is no imposed boundary along the material curve: it is a marked set of fluid particles, not a rigid obstacle or a wall.

## 5 The normalized Fibonacci matrix inside the same family

Let  $A_\Omega = \tau_\Omega M$ , as in the supplied golden-norm dossier. The spectral projectors of  $K$  are  $P_\pm = (I_2 \pm K)/2$ , so

$$e^{\eta K} = e^\eta P_+ + e^{-\eta} P_-.$$

The eigenvalues of  $M$  are  $\varphi$  and  $\varphi' = -\varphi^{-1}$ . It follows that

$$e^{2(\log \varphi)K} = \varphi^2 P_+ + \varphi^{-2} P_- = M^2. \quad (10)$$

**Corollary 5.1** (A continuous fluid realization of two matrix steps). *Choose a smooth dimensionless schedule  $\sigma(t)$  and set*

$$c(t) = \tau_\Omega^{2\sigma(t)}, \quad \eta(t) = 2\sigma(t) \log \varphi.$$

*At  $\sigma = 1$ , the material map is*

$$\Phi(w, z) = (A_\Omega^2 w, \tau_\Omega^{-4} z). \quad (11)$$

*For  $V_+(w) = 1 + \ell_+(w)$ , the entire path satisfies*

$$V_+(t) = 1 + \lambda^{2\sigma(t)}(V_+(0) - 1), \quad \mathcal{N}(w(t)) = \tau_\Omega^{4\sigma(t)} \mathcal{N}(w(0)), \quad z(t)\mathcal{N}(w(t)) = z(0)\mathcal{N}(w(0)).$$

*Proof.* Apply (10) and (5)–(6);  $ce^\eta = (\tau_\Omega \varphi)^{2\sigma} = \lambda^{2\sigma}$ .  $\square$

The square is intentional:  $\det A_\Omega = -\tau_\Omega^2 < 0$ , whereas a smooth planar flow map has positive determinant. Our plane-invariant flow realizes  $A_\Omega^2$ . A single  $A_\Omega$  remains the original discrete orientation-reversing map.

The similarity case  $\eta = 0$  preserves the physical shape and its Euclidean length ratios. The anisotropic case generally turns circles into ellipses, and does not preserve that same Euclidean shape. Their unity is the explicitly defined family (3), not an assertion that these two actions coincide. When  $c = 1$ ,  $\eta = 2 \log \varphi$ , the family instead realizes  $M^2$  on the plane and preserves  $\mathcal{N}$  itself.

## 6 Finite energy and smooth compact forcing

**Theorem 6.1** (Localized realization of a prescribed finite geometric motion). *Let  $T > 0$ , let  $c, \eta \in C^\infty(\mathbb{R})$  with  $c > 0$ , and suppose  $c = 1$ ,  $\eta = 0$  near and before  $t = 0$ , while  $c, \eta$  are constant near and after  $t = T$ . Let  $\mathcal{K} \subset \mathbb{R}^3$  be compact. For every  $\nu > 0$ , there exist globally smooth fields  $u, p$  and  $f \in C_c^\infty(\mathbb{R}^3 \times (0, \infty); \mathbb{R}^3)$  satisfying (7), such that:*

1.  $u(\cdot, 0) = 0$ , and  $u, p$  have compact spatial support and vanish outside a compact subinterval of  $(0, T)$ ;
2.  $\sup_{t \geq 0} \int_{\mathbb{R}^3} |u|^2 dX < \infty$ ;
3. each initial point  $X_0 \in \mathcal{K}$  follows exactly  $X(t) = \Phi_{c(t), \eta(t)}(X_0)$ ;
4.  $f$  vanishes in a neighborhood of the complete marked trajectory set; the forcing is confined to a surrounding transition region.

Consequently, Corollaries 4.1 and 5.1 hold for these marked particles in finite-energy, smoothly forced Navier–Stokes solutions.

*Proof.* The set  $\mathcal{T} = \{\Phi_{c(t), \eta(t)}(X_0) : X_0 \in \mathcal{K}, 0 \leq t \leq T\}$  is compact by continuity. Choose a smooth spatial cutoff  $\chi$  with compact support, equal to one on an open neighborhood of  $\mathcal{T}$ . For the affine fields of Theorem 3.1, define

$$\mathcal{A}(X, t) = \frac{1}{3}(u_{\text{aff}}(X, t) \times X), \quad u = \text{curl}(\chi \mathcal{A}), \quad p = \chi p_{\text{aff}}. \quad (12)$$

The vector identity for the curl of a cross product gives

$$\text{curl}(u_{\text{aff}} \times X) = u_{\text{aff}} \text{div} X - X \text{div} u_{\text{aff}} + (X \cdot \nabla) u_{\text{aff}} - (u_{\text{aff}} \cdot \nabla) X = 3u_{\text{aff}}.$$

Thus  $u = u_{\text{aff}}$  wherever  $\chi = 1$ , and  $\text{div} u = 0$  everywhere. Set

$$f = \partial_t u + (u \cdot \nabla) u - \nu \Delta u + \nabla p. \quad (13)$$

All fields in this formula are smooth with compact spatial support. Since  $B$  and all its derivatives vanish outside a compact time subinterval of  $(0, T)$ , the velocity, pressure, and force have the required time support. The force is identically zero wherever  $\chi = 1$  by Theorem 3.1, and wherever  $\chi$  is identically zero. It is therefore supported in the fixed transition region outside the marked trajectories.

The smooth velocity is uniformly bounded and compactly supported in space-time, giving a uniform finite-energy bound directly. By construction (7) holds exactly. Each proposed marked curve solves its particle ODE in the region where  $u = u_{\text{aff}}$ ; uniqueness for the smooth ODE identifies it with the actual global trajectory.  $\square$

**Explicit cutoffs.** Nothing depends on an unspecified discontinuous cutoff. Put  $b(s) = e^{-1/s}$  for  $s > 0$  and  $b(s) = 0$  for  $s \leq 0$ , and  $H_0(s) = b(s)/(b(s) + b(1 - s))$ . Then  $H_0$  is smooth, zero for  $s \leq 0$  and one for  $s \geq 1$ . For  $0 < t_0 < t_1 < T$ ,  $\sigma(t) = H_0((t - t_0)/(t_1 - t_0))$  gives an admissible schedule. Choose  $R_0 < R_1$  with  $\mathcal{T}$  strictly inside the ball of radius  $R_0$ ; then

$$\chi(X) = 1 - H_0\left(\frac{|X|^2 - R_0^2}{R_1^2 - R_0^2}\right)$$

is a suitable spatial cutoff.

**Remark 6.2** (Scope of the existence result). The fields and force are designed together. This proves a geometric realization theorem for smooth forced flows. It does not claim the same motion for an independently prescribed force. The path has finite duration, bounded deformation, and positive  $c$ ; no singular endpoint is included. The global fields are explicitly smooth for all time.

## 7 A vortical similarity variant

The preceding golden family is a strain flow. A separate variant adds actual rotation while preserving the marked planar shape.

**Proposition 7.1** (Swirl, strain, and its exact force). *For smooth rates  $\gamma, \omega$ , let*

$$B_s = \begin{pmatrix} \gamma & -\omega & 0 \\ \omega & \gamma & 0 \\ 0 & 0 & -2\gamma \end{pmatrix}, \quad u_s = B_s X, \quad p_s = -\frac{1}{2} X^T \text{sym}(\dot{B}_s + B_s^2) X.$$

*Then  $\text{div } u_s = 0$ ,  $\text{curl } u_s = 2\omega e_z$ , and*

$$\partial_t u_s + (u_s \cdot \nabla) u_s - \nu \Delta u_s + \nabla p_s = (\dot{\omega} + 2\gamma\omega)(-y, x, 0).$$

*If  $\gamma = \dot{c}/c$ , the particle map is  $(w, z) \mapsto (cR_{\theta}w, c^{-2}z)$ , with  $\dot{\theta} = \omega$  and identity initial conditions. It preserves the OMEGA length ratio. The core force vanishes precisely when  $(c^2\omega)' = 0$ .*

*Proof.* The symmetric part of  $\dot{B}_s + B_s^2$  has diagonal entries  $\dot{\gamma} + \gamma^2 - \omega^2$  twice and  $-2\dot{\gamma} + 4\gamma^2$ . The remaining skew part is  $(\dot{\omega} + 2\gamma\omega)$  times the planar rotation generator. This proves the equation and direct differentiation gives the curl and particle map. Rotation and uniform planar scaling preserve all marked length ratios.  $\square$

For a start-from-rest realization with nonzero later swirl, choose smooth compactly supported rates and retain the displayed core force. The curl localization still gives smooth compact forcing and finite energy, but the force need not be confined to the collar. If the core force is zero and  $\omega$  initially vanishes,  $(c^2\omega)' = 0$  prevents it from spontaneously becoming nonzero. Generic rotations do not preserve the golden form  $\mathcal{N}$ ; this proposition is the shape-preserving vortical branch.

## 8 Transport to the scalar, integer, and spiral branches

### 8.1 The frozen exact encoder

For positive scales  $s_i$ , retain the supplied coordinatewise maps

$$E_s(Y) = (\lfloor Y_i/s_i \rfloor, Y_i - s_i \lfloor Y_i/s_i \rfloor)_i, \quad D_s(d, \rho) = (s_i d_i + \rho_i)_i.$$

Their inverse identities imply, for every specified geometric map  $\Phi$ ,

$$\widehat{\Phi} = E_s \Phi D_s, \quad D_s \widehat{\Phi}^{\circ n} E_s = \Phi^{\circ n}.$$

This is an exact coordinate statement about the full real state. A finite precision implementation needs its own representation and validation contract. It is not an encoding of an arbitrary entire fluid field by a finite byte list.

### 8.2 The certified floor ledger

On the similarity branch, take a planar linear observation  $L(w) = hd$ ,  $h > 0$ ,  $d \in \mathbb{Z}_{\geq 0}$ . A  $c = \lambda$  step gives

$$\left\lfloor \frac{L(w')}{h} \right\rfloor = \lfloor \lambda d \rfloor.$$

The supplied independent arithmetic certificate establishes, for  $0 \leq d < 42189065$ ,

$$\lfloor \lambda d \rfloor = \left\lfloor \frac{Qd}{S} \right\rfloor, \quad Q = 339949771344778, \quad S = 10^{15}.$$

Keeping  $\rho = Qd - Sd'$  with  $0 \leq \rho < S$  gives the exact inverse  $d = (Sd' + \rho)/Q$  whenever the division is integral. This reproduces the existing arithmetic branch at a specified observation of the flow.

To iterate this integer map, quantization must be part of each state update. Rounding one uninterrupted continuous trajectory only at its final time is a different operation. For the exact rational stand-in,

$$F(F(26)) = 2, \quad \lfloor (Q/S)^2 26 \rfloor = 3.$$

The ledger certifies the deliberately defined integer process, not every unobserved fluid degree of freedom. TreeBABEL's step-31/32 result remains a theorem about its supplied integer alphabet and tick rule; no fluid stability threshold follows from those step numbers.

### 8.3 Sacks and golden-parameter observations

Keep the supplied definitions

$$\begin{aligned} g(k) &= k^2 - k - 1, \quad Z(t) = \sqrt{t} e^{2\pi i \sqrt{t}}, \quad \mathcal{W}(z) = e^{-2\pi i |z|} z, \\ \mathcal{R}_c(z) &= c e^{2\pi i (c-1)|z|} z, \quad \mathcal{K}_c(k) = \frac{1}{2} + \sqrt{c^2 g(k) + 5/4}, \quad k \geq \varphi. \end{aligned}$$

The existing identities give

$$\mathcal{W} \mathcal{R}_c = c \mathcal{W}, \quad \mathcal{R}_c Z(t) = Z(c^2 t), \quad g \mathcal{K}_c = c^2 g.$$

They share the planar scale  $c$  with our material-similarity subgroup. For example, its motion on a fixed nonnegative ray in the unwound plane becomes  $\mathcal{R}_c$  when displayed in Sacks coordinates. These are exact representation maps. Ulam integer tracks, prime-divisibility classes, and the Fibonacci/Lucas polynomial discriminants remain the supplied arithmetic statements; continuous fluid transport does not preserve integer labels.

### 8.4 The physical Navier–Stokes scaling law

An independent, standard equation symmetry must be distinguished from particle deformation. If  $(u, p, f)$  solves (7), define for  $c > 0$

$$u_c(X, t) = c^{-1} u(X/c, t/c^2), \quad p_c(X, t) = c^{-2} p(X/c, t/c^2), \quad f_c(X, t) = c^{-3} f(X/c, t/c^2).$$

The transformed fields solve the same equation with the same viscosity. Indeed each momentum term acquires  $c^{-3}$  and the divergence acquires  $c^{-2}$ . This maps one solution to another and rescales time. It is not the volume-preserving motion of one parcel in a fixed solution.

Likewise, using the nonlinear Sacks map as a spatial coordinate change requires transporting the differential operators and metric. On  $r > 0$ , the three-dimensional untwisting map  $(r, \theta, z) \mapsto (r, \theta - 2\pi r, z)$  pulls Euclidean length back to

$$ds^2 = dr^2 + r^2(d\theta - 2\pi dr)^2 + dz^2.$$

The connection, divergence, and Laplacian must be those of this metric in the transformed equations. The global homeomorphism used for the original spiral theorem alone is insufficient to justify a smooth PDE change of coordinates through the axis. The Sacks display uses dimensionless coordinates. For a physical radial coordinate with length units, first choose a reference length  $\ell_0$  and replace the twist  $2\pi r$  by  $2\pi r/\ell_0$ , and  $2\pi dr$  in the metric by  $(2\pi/\ell_0)dr$ .

## 9 Golden norm, fluctuation stress, and exact cubature

**Proposition 9.1** (An exact quadratic observation of a fluctuation stress). *Let  $v = \bar{v} + w$  be a planar random or phase-dependent vector with finite second moment,  $\langle w \rangle = 0$ , and let  $R = \langle ww^T \rangle$ . Then*

$$\boxed{\langle \mathcal{N}(v) \rangle - \mathcal{N}(\bar{v}) = \text{tr}(HR)}. \quad (14)$$

Moreover,

$$|\text{tr}(HR)| \leq \frac{\sqrt{5}}{2} \text{tr} R.$$

Under the normalized Fibonacci transformation  $v \mapsto A_\Omega v$ , the covariance becomes  $A_\Omega R A_\Omega^T$  and its golden observation becomes  $-\tau_\Omega^2 \text{tr}(HR)$ .

*Proof.* Expand  $(\bar{v} + w)^T H(\bar{v} + w)$  and average. The cross terms vanish and  $\langle w^T H w \rangle = \text{tr}(HR)$ . Since  $H^2 = (5/4)I_2$ , its eigenvalues are  $\pm\sqrt{5}/2$ . For the positive semidefinite covariance  $R$ , the claimed bound follows by diagonalizing  $H$ . Finally,  $A_\Omega^T H A_\Omega = -\tau_\Omega^2 H$  proves the transformation law.  $\square$

For a three-dimensional velocity fluctuation, this identity applies to its two planar components and the corresponding covariance block. It is an exact algebraic observation of Reynolds stress. It neither recovers the fluctuation field from its covariance nor asserts that a required fluid stress lies in the external manuscript's admissible stress cone.

**Proposition 9.2** (Cubature of the affine fluid observables). *Suppose nodes  $\omega_j \in S^2$  and weights  $w_j$  integrate spherical polynomials of degree at most two against surface area measure, so  $\sum_j w_j = 4\pi$ . For an affine core  $u(X) = BX$ , at radius  $r$ ,*

$$\sum_j w_j |u(r\omega_j)|^2 = \frac{4\pi r^2}{3} \text{tr}(B^T B), \quad (15)$$

$$\sum_j w_j u(r\omega_j) u(r\omega_j)^T = \frac{4\pi r^2}{3} B B^T. \quad (16)$$

Pressure from (8) is also integrated exactly. The kinetic energy of the affine core in a ball of radius  $R_0$  is

$$\frac{1}{2} \int_{|X| \leq R_0} |BX|^2 dX = \frac{2\pi R_0^5}{15} \text{tr}(B^T B).$$

*Proof.* Rotational symmetry and  $\sum_i \omega_i^2 = 1$  give  $\int_{S^2} \omega \omega^T d\sigma = (4\pi/3)I_3$ . Each integrand is a polynomial of degree two in  $\omega$ , so the assumed cubature is exact. Integrating  $r^4$  from 0 to  $R_0$  proves the last formula.  $\square$

This supplies a concrete use of the spherical-sampling branch: exact angular energy, pressure, and stress measurements for the affine core. It does not assert exact integration of the cutoff transition region by the same fixed degree rule. Equal weights on the 12 normalized icosahedral vertices already suffice for the moments used here; the companion script checks them exactly. The previously reported higher-degree cubature certificates are not needed for this proposition and were not rerun for this extension.

## 10 Assembly and the relation to the external blowup claim

**Theorem 10.1** (OMEGA-compatible geometric fluid extension). *The supplied geometry, golden algebra, contraction, and frozen exact encoder extend to the family (3) with the following proved connections:*

1. *Its similarity subgroup transports a material OMEGA construction and preserves  $(G + C)/a = \Omega_0$ ; its scalar observation is the OMEGA contraction.*
2. *Its golden anisotropic subgroup realizes  $M^2$  and  $A_\Omega^2$ , with the observation and norm laws in Corollary 5.1.*
3. *The full family preserves three-dimensional volume and the cubic material invariant  $z\mathcal{N}(w)$ .*
4. *Every smooth finite motion with stationary endpoints has the finite-energy, smooth compact-force realization of Theorem 6.1.*
5. *Exact encoding transports these finite-dimensional material states without altering SRSD. The certified floor ledger applies to the specified quantized scalar observation.*
6. *The same scale parameter has the existing Sacks/golden-parameter representations; golden quadratic stress and affine cubature identities hold as stated in Propositions 9.1 and 9.2.*

*Proof.* Each item is the indicated theorem or proposition. Composition is only on the stated domains. Material states, velocity fields, integer samples, covariances, and geometric display coordinates remain different types.  $\square$

| Existing branch                  | Added exact connection                                               | Intended site location          |
|----------------------------------|----------------------------------------------------------------------|---------------------------------|
| Marked geometry and $\Omega_0$   | Material planar similarity; constant normalized length               | geometry, omega0                |
| Contraction                      | $V \circ \Phi_{\lambda,0} = T \circ V$                               | dynamics                        |
| Golden norm and matrix           | Volume-preserving lift; invariant $z\mathcal{N}$ ; $A_\Omega^2$ step | network plus supplied extension |
| Rational floor ledger            | Explicit scalar grid observation with per-step quantization          | dynamics                        |
| Frozen SRSD                      | Exact encoding of the specified material state                       | srsgd                           |
| Sacks and norm-polynomial tracks | Coordinate representations using the same $c$                        | Supplied golden-norm dossier    |
| Icosahedral cubature             | Exact degree-two affine energy and stress measurements               | cubature                        |
| Verification contract            | Named hypotheses and exact symbolic identities                       | contract                        |

### 10.1 What the fluid blowup manuscript contributes to this discussion

The external manuscript [3] claims smooth compact-force Navier–Stokes blowup via a collapsing core and oscillatory stress corrections. Its Theorem 1.1 and relevant mechanism sections were inspected; its complete proof and Lean build were not independently verified here. Its similarity scales describe a concentrating flow region, not the volume of a fixed material parcel. Incompressible particle transport still preserves volume.

Our construction provides an exact regular geometric embedding. It does not identify its affine core with that manuscript’s singular vortex, nor inherit its blowup conclusion. In particular, the finite localization proof uses a compact trajectory set and smooth bounded rates. Those hypotheses would have to be reconsidered at a singular endpoint. The external manuscript’s smooth residual near blowup requires derivative estimates and stress realization that this extension does not supply.

The comparison does establish a precise common discipline: specify the state, the observation, and the full residual of the equation. In the integer ledger, the residual reconstructs a predecessor. In covariance,

it records a quadratic effect of unresolved variation. In (13), it is an actual force field. These objects have different domains and different validity tests.

## 10.2 What is forced to remain constant

For the shape-preserving branch, the geometric identity (1) and the proved fluid transport force the readout to remain  $\Omega_0$ . For the golden family, the conserved material scalar is  $J = z\mathcal{N}(w)$ . These are conditional invariants of explicitly constructed flows. Navier–Stokes has not independently selected  $\varphi + \pi$ : its value enters through the initial geometric construction and the chosen rates. The result is an exact realization and preservation theorem with those choices visible.

## 11 Verification and reproducibility

The companion `verify_omega_ns.py` uses exact SymPy algebra and explicit exceptions. Its checks cover matrix generators, the group law, determinant, golden observations, the cubic first integral, every term of the affine momentum equation, the particle ODE, the curl cutoff’s divergence, the two-step Fibonacci realization, the swirl force, the stress observation, and icosahedral moments. It also checks the integer counterexample separating iterated quantization from final rounding.

```
python verify_omega_ns.py
python -O verify_omega_ns.py
```

The ordinary and optimized runs passed 34 exact checks with identical output, using SymPy 1.14.0. These computations support the algebra. The written proof establishes smooth cutoff existence, compactness, bounded energy, and the general localization result; these are not inferred from a simulation. No numerical Navier–Stokes evolution, new singularity, or Lean certificate is claimed.

The source synthesis’s prior corrections remain in force: integer sampling is required in its Sacks finite-error statement; integer scale, remainder, and byte hypotheses are required for its streaming quotient bound; and the original pasted decoder’s fail-closed claim is not used by this extension. No existing source file or frozen foundation has been overwritten.

## References

- [1] Matthew Alan Newman, supplied OMEGA thread and archive index; *OMEGA Golden Norm and Spiral Transport*, version 1.0, supplied TeX/PDF; *OMEGA: Exact-State Transport*, synthesis 1.0, 17 September 2026; and its accompanying claim-check corrections. Website integration target: <https://omega-naught.grok.me/>. Current website content was inaccessible in this session.
- [2] Charles L. Fefferman, *Existence and Smoothness of the Navier–Stokes Equation*, Clay Mathematics Institute problem statement. <https://www.claymath.org/wp-content/uploads/2022/06/navierstokes.pdf>. Used for the equation and finite-energy setting, not as a source of the OMEGA-specific construction proved here.
- [3] OpenAI, *Finite Time Blowup for Navier–Stokes*, released September 2026, 166-page manuscript, Theorem 1.1 and Sections 2–3, 5, 10. <https://cdn.openai.com/pdf/32d9f210-8b73-45e0-91bc-82a30aef8a9a/navier-stokes.pdf>. An external claimed result, not a dependency of the geometric existence theorems in this extension.