

The Omega–Alpha Framework

Affine Contraction, Exact Quotient–Remainder Witnesses,
and Deterministic Multiscale Geometric Probes

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Abstract

This paper consolidates a sequence of earlier preprints into a single mathematically scoped framework. Let

$$\phi = \frac{1 + \sqrt{5}}{2}, \quad \Omega_0 = \phi + \pi, \quad \lambda_\Omega = \frac{\phi}{\phi + \pi}.$$

The continuous layer is the affine homothety

$$\Omega_\alpha(x) = \alpha + \lambda_\Omega(x - \alpha),$$

which is a strict contraction with fixed point α , closed-form iterates, an exact symbolic inverse, and a continuous semigroup representation. The finite layer fixes a scale S , defines $Q = \lfloor S\lambda_\Omega \rfloor$, and applies

$$d_{k+1} = \lfloor Qd_k/S \rfloor, \quad \rho_k = Qd_k - Sd_{k+1}.$$

The quotient map alone is many-to-one, but the terminal value together with the complete ordered Euclidean-remainder ledger reconstructs the initial integer exactly. This boundary-witness theorem extends componentwise to finite digital states and supplies a deterministic audit and replay mechanism after quantization.

The same scale coefficient defines a local radial map on a Riemannian surface through the exponential and logarithm maps. Combined with a fixed angular aperture, this yields a deterministic multiscale geometric probe: a prescribed sequence of scales at which a declared measurement functional is evaluated and, after quantization, exactly witnessed. Finite applications are given for 15-bit transport, monotone byte and 16-bit codebooks, a finite coordinate for Ramanujan’s ellipse-perimeter multiplier, conservative collision screening, and scalar logical ordering. Each application is explicitly separated from the foundational theorems. The paper also records exhaustive verification results, implementation conventions, and corrections to earlier terminology. No claim is made that the construction is a universal physical law, a generic compression method, or a cryptographic primitive without an independent security definition.

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1 Introduction

The work synthesized here began with a geometric normalization involving the golden ratio and the circle constant, developed into an affine contraction, and was later implemented as an exact integer quotient–remainder process in the author’s Figshare archive [19, 20, 21]. Earlier drafts used several overlapping names and sometimes attached physical, cryptographic, or computational interpretations that were stronger than the mathematics supported. The present paper separates the construction into four levels:

1. an exact symbolic coefficient and affine contraction;
2. a finite rational realization and exact ordered witness;
3. deterministic multiscale sampling built on the contraction;
4. applications whose validity depends on additional domain assumptions.

The principal exact result is elementary but useful: every remainder discarded by a floor contraction is retained in an ordered ledger, and the ledger supplies an exact left inverse on valid encoded states. The contribution is therefore not a claim that Euclidean division or affine contraction is new. It is a particular integrated specification: a reproducibly derivable scale coefficient, a fixed finite arithmetic profile, a strict witness format, and a family of multiscale uses.

1.1 Contributions

The paper establishes the following.

- The coefficient $\lambda_\Omega = \phi/(\phi + \pi)$ lies in $(0, 1)$ and is transcendental.
- The map $\Omega_\alpha(x) = \alpha + \lambda_\Omega(x - \alpha)$ is a strict affine contraction on a real normed vector space, with exact iterates, inverse, and continuous semigroup.
- The local Riemannian map $p \mapsto \exp_\alpha(\lambda_\Omega \log_\alpha p)$ gives exact radial contraction inside a normal neighborhood.
- For any positive integer scale S , the finite coefficient $Q = \lfloor S\lambda_\Omega \rfloor$ satisfies a sharp one-unit scaled error bound.
- The terminal quotient and complete ordered Euclidean-remainder ledger determine the initial nonnegative integer exactly.
- A fixed multiscale measurement protocol can be made replayable by witnessing its quantized observations and decisions.
- The declared 15-bit transport and two finite codebooks satisfy the reported exhaustive tests.

1.2 Scope

Exact reconstruction begins at the declared digital boundary. A witness cannot recover information already discarded by a sensor, an external numerical approximation, or an input quantizer. Likewise, a deterministic hash binds bytes under a serialization rule but does not by itself authenticate an author. The construction is not presented as a universal physical model, a proof about quantum mechanics or cosmology, or an optimization theorem over all possible contraction coefficients.

2 Constants, notation, and arithmetic status

Definition 2.1 (Omega coefficient). Let

$$\phi = \frac{1 + \sqrt{5}}{2}, \quad \Omega_0 = \phi + \pi,$$

and define

$$\lambda_\Omega = \frac{\phi}{\Omega_0} = \frac{\phi}{\phi + \pi}.$$

Because $\phi > 0$ and $\pi > 0$, one has $0 < \lambda_\Omega < 1$. Numerically,

$$\lambda_\Omega = 0.339949771344778929576702809321279 \dots$$

The displayed decimal is an approximation; the quotient above is the exact symbolic definition.

Proposition 2.2 (Transcendence). *The number λ_Ω is transcendental.*

Proof. The golden ratio ϕ is algebraic. Suppose that λ_Ω were algebraic. Since $\lambda_\Omega \neq 0$,

$$\pi = \phi \frac{1 - \lambda_\Omega}{\lambda_\Omega}$$

would then be algebraic, because algebraic numbers form a field. This contradicts the transcendence of π [2]. Hence λ_Ω is transcendental. $\square$

Remark 2.3 (What the transcendence result does not imply). Transcendence does not imply cryptographic unpredictability, statistical randomness, optimal Diophantine behavior, or absence of finite cycles after quantization. Those are separate properties and require separate definitions and proofs.

Define the decay rate

$$\kappa = -\log \lambda_\Omega = 1.07895740362416456641369720327285 \dots,$$

so that $e^{-\kappa} = \lambda_\Omega$.

3 The affine Omega–Alpha contraction

Let X be a real normed vector space and fix $\alpha \in X$.

Definition 3.1 (Omega–Alpha map). For $x \in X$, define

$$\Omega_\alpha(x) = \alpha + \lambda_\Omega(x - \alpha).$$

Equivalently,

$$\Omega_\alpha(x) = \lambda_\Omega x + (1 - \lambda_\Omega)\alpha = \frac{\phi}{\phi + \pi}x + \frac{\pi}{\phi + \pi}\alpha.$$

Thus $\Omega_\alpha(x)$ is the convex combination of x and α with the declared weights.

Theorem 3.2 (Contraction and fixed point). *For all $x, y \in X$,*

$$\|\Omega_\alpha(x) - \Omega_\alpha(y)\| = \lambda_\Omega \|x - y\|.$$

Consequently, Ω_α is a strict contraction. Its unique fixed point is α .

Proof. Direct subtraction gives

$$\Omega_\alpha(x) - \Omega_\alpha(y) = \lambda_\Omega(x - y),$$

which proves the norm identity. Also $\Omega_\alpha(\alpha) = \alpha$. If $\Omega_\alpha(x) = x$, then $(1 - \lambda_\Omega)(x - \alpha) = 0$. Since $\lambda_\Omega \neq 1$, one obtains $x = \alpha$. $\square$

Theorem 3.3 (Closed-form iteration). *For every $n \in \mathbb{N}$ and $x \in X$,*

$$\Omega_\alpha^n(x) = \alpha + \lambda_\Omega^n(x - \alpha).$$

In particular,

$$\|\Omega_\alpha^n(x) - \alpha\| = \lambda_\Omega^n \|x - \alpha\| \quad \text{and} \quad \Omega_\alpha^n(x) \longrightarrow \alpha.$$

Proof. The formula follows by induction. The convergence follows from $0 < \lambda_\Omega < 1$. $\square$

Theorem 3.4 (Exact symbolic inverse). *Define*

$$\Xi_\alpha(y) = \alpha + \frac{y - \alpha}{\lambda_\Omega}.$$

Then $\Xi_\alpha \circ \Omega_\alpha$ and $\Omega_\alpha \circ \Xi_\alpha$ are the identity on X . More generally,

$$\Xi_\alpha^n(y) = \alpha + \lambda_\Omega^{-n}(y - \alpha).$$

Remark 3.5 (Symbolic inverse versus implemented recovery). The inverse theorem is exact over the mathematical field in which the expression is evaluated. It does not state that a finite floating-point value rounded near α contains enough information to recover its preimage; standard floating-point error and reproducibility issues are discussed in [4, 5]. Once two inputs have rounded to the same machine value, the symbolic inverse cannot distinguish them without auxiliary information.

3.1 Continuous semigroup

Definition 3.6 (Continuous Omega-Alpha flow). For $t \geq 0$, define

$$\Omega_\alpha(t, x) = \alpha + e^{-\kappa t}(x - \alpha).$$

Proposition 3.7 (Semigroup and differential equation). *For $s, t \geq 0$,*

$$\Omega_\alpha(t + s, x) = \Omega_\alpha(t, \Omega_\alpha(s, x)).$$

Moreover, $z(t) = \Omega_\alpha(t, x)$ is the unique solution of

$$\dot{z}(t) = -\kappa(z(t) - \alpha), \quad z(0) = x.$$

The unit-time map is $\Omega_\alpha(1, x) = \Omega_\alpha(x)$.

3.2 Geometric classification

In $\mathbb{R}^m$, Ω_α is a positive affine homothety centered at α . Its fixed-point behavior is a direct special case of the contraction principle [1]. It preserves lines, parallelism, orientation, angles, and similarity classes. Lengths are multiplied by λ_Ω , j -dimensional contents by λ_Ω^j , and m -dimensional Lebesgue measure by λ_Ω^m . These are standard consequences of the derivative $D\Omega_\alpha = \lambda_\Omega I$.

4 Geometric source construction

The source diagrams place a center α in a unit-circle construction, use a golden-ratio linear extension, and distinguish two nearby radial directions by one arcsecond. The rigorous content is stated here without assigning physical necessity to the diagram.

Let $r > 0$, and let two unit vectors u_- and u_+ in the Euclidean tangent plane make the angle

$$\theta = 1 \text{ arcsecond} = \frac{\pi}{648000} \text{ radians.}$$

Set

$$d = \alpha + ru_-, \quad b = \alpha + ru_+.$$

Then the circular arc between the two rays on the radius- r circle has length $r\theta$, while the chord length is

$$\|b - d\| = 2r \sin(\theta/2).$$

The golden extension identity is

$$r + \phi r = (1 + \phi)r = \phi^2 r.$$

The combined scalar $(\phi + \pi)r$ and the normalized golden part

$$\frac{\phi r}{(\phi + \pi)r} = \lambda_\Omega$$

are definitions motivated by these declared linear and circular quantities. The diagram does not prove that all geometry uniquely determines this ratio; it specifies the ratio used by the framework.

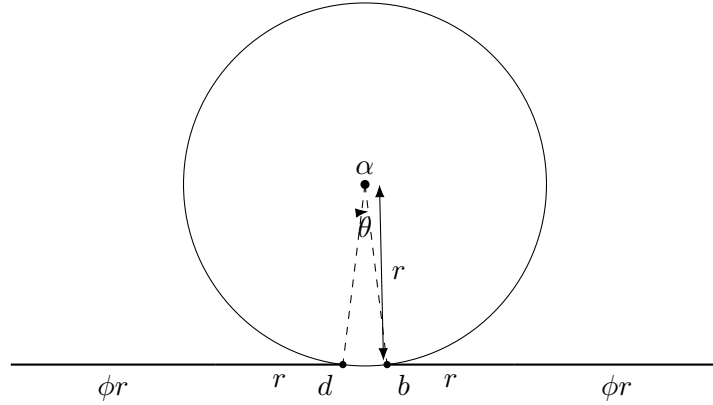


Figure 1: Schematic formalization of the source construction. The drawn angle is exaggerated; the declared aperture is one arcsecond.

5 Local extension to curved two-dimensional surfaces

Let (M, g) be a smooth two-dimensional Riemannian manifold and let $\alpha \in M$; standard background on normal neighborhoods and exponential coordinates may be found in [8, 9]. Denote the exponential map by $\exp_\alpha$, and let U be a normal neighborhood on which $\log_\alpha = \exp_\alpha^{-1}$ is single-valued.

Definition 5.1 (Local Riemannian Omega map). For $p \in U$, define

$$\Omega_\alpha^M(p) = \exp_\alpha(\lambda_\Omega \log_\alpha(p)).$$

Theorem 5.2 (Exact radial scaling). *Suppose $p = \exp_\alpha(v) \in U$ and the radial geodesic $t \mapsto \exp_\alpha(tv)$, $0 \leq t \leq 1$, is minimizing. Then*

$$d_g(\alpha, \Omega_\alpha^M(p)) = \lambda_\Omega d_g(\alpha, p).$$

For every $n \geq 0$ for which the points remain in the normal neighborhood,

$$(\Omega_\alpha^M)^n(p) = \exp_\alpha(\lambda_\Omega^n \log_\alpha(p)).$$

Proof. The radial geodesic has constant speed $\|v\|_g$, so the point $\exp_\alpha(tv)$ lies at distance $t\|v\|_g$ from α while the geodesic is minimizing. Set $t = \lambda_\Omega$. Iteration follows by uniqueness of the logarithm on U . $\square$

This is a radial statement. On a general curved manifold it does not follow that

$$d_g(\Omega_\alpha^M(p), \Omega_\alpha^M(q)) = \lambda_\Omega d_g(p, q)$$

for arbitrary p, q . Curvature controls the transverse response.

5.1 Fixed-aperture surface probe

Choose unit vectors $u_-, u_+ \in T_\alpha M$ with angle $\theta = \pi/648000$, and let

$$p_n^- = \exp_\alpha(\lambda_\Omega^n r_0 u_-), \quad p_n^+ = \exp_\alpha(\lambda_\Omega^n r_0 u_+).$$

The radial scale $r_n = \lambda_\Omega^n r_0$ is exactly prescribed, while the geodesic separation

$$c_n = d_g(p_n^-, p_n^+)$$

and the arc length along the geodesic circle are surface-dependent measurements. In geodesic polar coordinates, the metric may be written locally as

$$ds^2 = dr^2 + G(r, \psi)^2 d\psi^2,$$

and the fixed-radius arc response is

$$A(r, \theta) = \int_{-\theta/2}^{\theta/2} G(r, \psi) d\psi.$$

For small r , the expansion

$$G(r, \psi) = r - \frac{K(\alpha)}{6} r^3 + O(r^4)$$

shows how Gaussian curvature first perturbs the Euclidean response. Thus λ_Ω supplies the scale law; the d - b aperture supplies the local comparison; and the measured response records the surface geometry.

6 Finite rational realization

Definition 6.1 (Scaled rational coefficient). For an integer $S \geq 1$, define

$$Q = \lfloor S\lambda_\Omega \rfloor, \quad \lambda_S = \frac{Q}{S}.$$

Proposition 6.2 (Approximation bound).

$$0 \leq \lambda_\Omega - \lambda_S < \frac{1}{S}.$$

Proof. The defining inequality for the floor function is

$$Q \leq S\lambda_\Omega < Q + 1.$$

Divide by S . □

The reference protocol fixes

$$S = 10^{15}, \quad Q = 339949771344778, \quad \lambda_S = 0.339949771344778.$$

These integers, not an implementation's floating approximation, control the finite arithmetic.

7 Exact quotient–remainder witnesses

Let $d_k \in \mathbb{Z}_{\geq 0}$.

Definition 7.1 (One finite contraction step). Define

$$d_{k+1} = \left\lfloor \frac{Qd_k}{S} \right\rfloor, \quad \rho_k = Qd_k - Sd_{k+1}.$$

Lemma 7.2 (Euclidean decomposition). *For every step,*

$$Qd_k = Sd_{k+1} + \rho_k, \quad 0 \leq \rho_k < S.$$

The quotient and remainder are unique.

Proof. This is Euclidean division of Qd_k by the positive integer S . □

Proposition 7.3 (Visible contraction). *If $d_k > 0$, then $d_{k+1} < d_k$. If $d_k = 0$, then $d_{k+1} = 0$.*

Proof. Since $0 < Q < S$, one has $0 \leq Qd_k/S < d_k$ for $d_k > 0$. Taking the floor preserves the strict upper bound. □

Theorem 7.4 (One-step witnessed recovery). *For a forward-generated pair (d_{k+1}, ρ_k) ,*

$$d_k = \frac{Sd_{k+1} + \rho_k}{Q}.$$

Proof. Rearrange the Euclidean decomposition. Exact divisibility follows because the numerator equals Qd_k . □

7.1 The ordered boundary witness

Definition 7.5 (Boundary witness). Fix $N \geq 0$. Iterate

$$d_0 \mapsto d_1 \mapsto \cdots \mapsto d_N$$

and retain the ordered ledger

$$W_N = (\rho_0, \rho_1, \dots, \rho_{N-1}).$$

The complete boundary object is

$$\partial_{S,Q,N}(d_0) = (d_N, W_N, S, Q, N).$$

Theorem 7.6 (Exact boundary-witness reconstruction). *The map $d_0 \mapsto \partial_{S,Q,N}(d_0)$ is injective. Its image admits the exact reconstruction algorithm*

$$d_k = \frac{Sd_{k+1} + \rho_k}{Q}, \quad k = N-1, N-2, \dots, 0.$$

Proof. Starting from d_N , each reverse step is uniquely determined by the corresponding remainder and the one-step recovery theorem. Repeating the process produces a unique d_0 . Therefore two distinct initial values cannot have the same complete boundary object. $\square$

Theorem 7.7 (Global witness identity). *Every forward-generated N -step witness satisfies*

$$Q^N d_0 = S^N d_N + \sum_{k=0}^{N-1} \rho_k S^k Q^{N-1-k}.$$

Proof. For $N = 1$, this is $Qd_0 = Sd_1 + \rho_0$. Suppose the statement holds for N . Multiply by Q and substitute $Qd_N = Sd_{N+1} + \rho_N$:

$$\begin{aligned} Q^{N+1} d_0 &= S^N Qd_N + \sum_{k=0}^{N-1} \rho_k S^k Q^{N-k} \\ &= S^{N+1} d_{N+1} + S^N \rho_N + \sum_{k=0}^{N-1} \rho_k S^k Q^{N-k}, \end{aligned}$$

which is the required identity at depth $N + 1$. $\square$

Remark 7.8 (Order is structural). The power multiplying ρ_k depends on the step index. In general, permuting the ledger changes the global identity and the reconstructed path. The witness is therefore an ordered sequence, not a multiset.

Remark 7.9 (Correct reversibility terminology). The quotient map $d \mapsto \lfloor Qd/S \rfloor$ is many-to-one. The augmented encoding has a left inverse on its valid image. It is not an involution, because the encoder and decoder have different domains and codomains.

7.2 Validation of a received witness

A strict decoder should check:

1. the declared S, Q, N and ledger length;
2. integer types, rejecting Boolean values where languages treat them as integers;
3. $0 \leq \rho_k < S$ for every ledger entry;
4. exact divisibility of each reverse numerator by Q ;
5. complete forward replay of the reconstructed initial value;
6. optionally, the global identity as a redundant algebraic check.

The replay requirement rejects noncanonical or fabricated ledgers that happen to pass isolated divisibility tests.

8 Vectors, signed values, and 15-bit transport

The scalar witness extends componentwise to a finite vector $D = (d_1, \dots, d_m) \in \mathbb{Z}_{\geq 0}^m$. Each coordinate is encoded independently, and exact reconstruction follows coordinatewise.

Signed integers may be transported through any explicitly invertible signed-to-nonnegative mapping, for example the zigzag map

$$\zeta(z) = \begin{cases} 2z, & z \geq 0, \\ -2z - 1, & z < 0. \end{cases}$$

Its inverse is fixed by parity.

8.1 The fold15_be_lsbpad_v1 rule

The reference byte transport interprets a byte string as a big-endian, most-significant-bit-first bitstream. It appends between zero and fourteen zero bits at the least-significant end until the total length is divisible by 15, then partitions the stream into limbs in

$$\{0, 1, \dots, 2^{15} - 1\}.$$

The number of padding bits and the original byte length are part of the transport metadata. Decoding concatenates the limbs, verifies that the declared padding bits are zero, removes them, and restores exactly the original number of bytes.

Proposition 8.1 (Transport reversibility). *Under strict length and padding validation, folding followed by unfolding is the identity on every finite byte string.*

Proof. The forward operation appends known zero bits and partitions without changing bit order. The reverse operation concatenates the same limbs and removes exactly the declared suffix. The length check prevents alternate interpretations. $\square$

9 The status of the 23-step horizon

The reference profile uses

$$N = 23.$$

This value is a fixed protocol depth, not a universal maximum of mathematical reversibility.

Proposition 9.1 (Extinction bound on 15-bit limbs). *For the reference S, Q , every $d_0 \in \{0, \dots, 32767\}$ reaches zero within ten contraction steps.*

Proof. Let $r = Q/S$. By induction,

$$F^n(d_0) \leq \lfloor r^n d_0 \rfloor.$$

A direct exact-rational or certified decimal calculation gives

$$r^{10} 32767 < 1.$$

Hence $F^{10}(d_0) = 0$ for the entire domain. The value 32767 remains positive through step nine, so ten is attained. $\square$

Thus a 23-entry ledger contains deterministic trailing zero-state steps for this domain. Fixed length nevertheless has protocol value: it supplies one unambiguous record shape and prevents variable-length trailing-state aliases.

A separate declared guard-band criterion can also select 23. For example,

$$\lambda_{\Omega}^{23} = 1.669318967862 \dots \times 10^{-11} > 10^{-11},$$

while $\lambda_{\Omega}^{24} < 10^{-11}$. This makes 23 the greatest integer satisfying $\lambda_{\Omega}^n \geq 10^{-11}$, but only after the threshold 10^{-11} is adopted as a design requirement. The primality of 23 may be useful in a separate modular or periodic construction; primality alone does not establish this numerical horizon.

10 Specification-derived parameter generation

Earlier drafts used the word *autogenesis*. The mathematical content can be stated without implying spontaneous computation.

Definition 10.1 (Specification-derived parameter). Let Σ be a finite symbolic specification and S a declared finite scale. A parameter $Q_\Sigma(S)$ is specification-derived if it is uniquely determined by Σ, S , can be produced by a deterministic terminating algorithm, and is independent of the payload later processed by the protocol.

For

$$\Sigma_\Omega = \left\{ \phi = \frac{1 + \sqrt{5}}{2}, \pi, \lambda_\Omega = \frac{\phi}{\phi + \pi} \right\},$$

define

$$Q_\Omega(S) = \lfloor S\lambda_\Omega \rfloor.$$

Since ϕ and π are computable reals in the standard sense of computable analysis [3], certified rational intervals for λ_Ω can be refined until the lower and upper interval endpoints have the same scaled floor. At that point $Q_\Omega(S)$ has been determined exactly. The parameter therefore need not be transmitted as an unexplained lookup value, although a production implementation may still pin and verify it for efficiency and interoperability.

This is the rigorous sense in which the operator environment precedes the payload: the specification fixes the arithmetic substrate, and the data later supply the operands.

11 Finite monotone codebooks

Define

$$F(v) = \lfloor Qv/S \rfloor, \quad T_N(b) = F^N((b+2)S).$$

The reference codebooks use $N = 23$.

11.1 Analytic spacing bound

Write one step as $v_{k+1} = rv_k - e_k$, where $r = Q/S$ and $0 \leq e_k < 1$. After N steps,

$$T_N(b) = Sr^N(b+2) - E_b, \quad 0 \leq E_b < \sum_{j=0}^{N-1} r^j < \frac{1}{1-r}.$$

Consequently,

$$T_N(b+1) - T_N(b) = Sr^N - (E_{b+1} - E_b).$$

For the reference constants and $N = 23$, this confines the integer difference to

$$T_{23}(b+1) - T_{23}(b) \in \{16692, 16693, 16694\}.$$

Hence the map is strictly increasing on every consecutive integer seed domain for which the fixed-width arithmetic does not overflow.

11.2 Byte codebook

For $b \in \{0, \dots, 255\}$, exhaustive evaluation gives

$$T(0) = 33386, \quad T(65) = 1118443, \quad T(255) = 4290149.$$

Moreover,

$$T(b) = 16693(b+2) + \varepsilon_b, \quad 0 \leq \varepsilon_b \leq 48.$$

Since $48/16693 < 1/2$, the direct decoder

$$b = \left\lfloor \frac{T(b)}{16693} + \frac{1}{2} \right\rfloor - 2$$

is exact on the byte codebook. The SHA-256 digest of the compact JSON array is

0fc8922c2d84cfd965c9fe310070e00ffd86ed76117584a9ff2faa85136a0c47.

11.3 Full 16-bit codebook

For $b \in \{0, \dots, 65535\}$, exhaustive evaluation yields 65,536 distinct increasing ticks, from 33,386 through 1,094,021,571, with the same adjacent-difference range. A domain-specific decoder may repeat

$$t \leftarrow \lfloor tS/Q \rfloor$$

23 times, round t/S to obtain a candidate $b + 2$, and then require exact forward replay. The repeated expansion is an approximation, not the inverse of the floor contraction. The replay check is what makes accepted decoding exact on the declared codebook.

The direct intermediate product vQ exceeds unsigned 64-bit range. A fixed-width implementation therefore requires unsigned 128-bit intermediate arithmetic or a separately proved wide multiply-divide routine.

12 Deterministic multiscale geometric probes

The finite and continuous layers combine naturally into a general measurement protocol.

Definition 12.1 (Omega-Alpha multiscale probe). Fix a reference point or state α , an initial scale $r_0 > 0$, depth N , aperture or comparison parameter θ , a measurement functional M , a deterministic quantizer $\mathcal{Q}$, and a witness encoder $\mathcal{W}$. Define

$$r_n = \lambda_\Omega^n r_0, \quad m_n = M(\alpha, r_n, \theta), \quad q_n = \mathcal{Q}(m_n), \quad \omega_n = \mathcal{W}(q_n).$$

The ordered object

$$\mathcal{P}_{\Omega, \alpha} = [(r_n, q_n, \omega_n)]_{n=0}^N$$

is an Omega-Alpha multiscale geometric probe.

The word *geometric* refers to the prescribed similarity scale law and to the declared metric or measurement geometry. The target may be a physical curve or surface, a digital state space, a reachable set, a residual field, or another object with a specified measurement functional.

Proposition 12.2 (Replay property). *Assume that the scale-generation, measurement, quantization, and decision algorithms are deterministic and versioned, and that every finite input and result required by the computation is included in, or recoverable from, a valid boundary witness. Then the complete digital probe can be replayed exactly.*

Proof. The symbolic or fixed rational scale sequence is deterministic. The witness reconstructs each represented finite state exactly. Applying the same versioned deterministic algorithms to the reconstructed states reproduces the same quantized measurements and decisions. $\square$

The proposition concerns computational replay. It does not imply that the quantized measurements equal an inaccessible continuous state without measurement error.

13 Application I: finite ellipse-multiplier coordinate

For an ellipse with semiaxes $a \geq b > 0$, set $r = a/b$ and

$$h = \left(\frac{r-1}{r+1} \right)^2.$$

Ramanujan's second perimeter approximation [10, 12] may be written

$$P_R = \pi(a+b)w(r), \quad w(r) = 1 + \frac{3h}{10 + \sqrt{4-3h}}.$$

Define the affine coordinate

$$v(r) = 1 + \frac{w(r) - 1}{\lambda_\Omega}.$$

Then

$$w(r) = 1 + \lambda_\Omega(v(r) - 1)$$

and

$$P_R = \pi(a+b)[1 + \lambda_\Omega(v(r) - 1)].$$

The nonlinear multiplier is evaluated once and represented by v ; subsequent reconstruction of that stored multiplier is affine. This is not a new elementary formula for the exact ellipse perimeter. The exact perimeter remains expressible through the complete elliptic integral of the second kind [11].

Under the revised 15-decimal policy, the multiplier is first evaluated at higher precision and truncated, then the coordinate is computed from the stored multiplier using the declared literal 0.339949771344778 and truncated. Selected verified values appear in table 1.

Table 1: Corrected finite ellipse-coordinate values.

Aspect ratio	Stored multiplier w	Coordinate v
1	1.000000000000000	1.000000000000000
ϕ	1.013981238987019	1.041127366939274
2	1.027976282991305	1.082295342869730
3	1.063544374901302	1.186922834658579
5	1.114616532029377	1.337157255838047
10	1.175989279709379	1.517692007890454

Two asymptotic values must remain distinct. The exact line-segment geometric multiplier relative to $\pi(a+b)$ is $4/\pi$, while Ramanujan's formula tends to $14/11$. Their Omega coordinates are correspondingly different.

14 Application II: conservative multiscale collision screening

Let two objects have a declared deterministic relative-motion model over a time interval I , and let $R_H > 0$ be a fixed hard-body or exclusion radius. A multiscale screen should not contract the physical collision radius toward zero. Instead, contract a conservative margin toward the fixed boundary.

Choose $B_0 \geq R_H$ and define

$$B_n = R_H + \lambda_\Omega^n(B_0 - R_H).$$

Proposition 14.1 (Nested conservative thresholds). *For every $n \geq 0$,*

$$R_H \leq B_{n+1} \leq B_n, \quad B_n \longrightarrow R_H.$$

If a true conjunction satisfies $d_{\min} \leq R_H$, then it satisfies $d_{\min} \leq B_n$ at every level.

Proof. The first statement follows from $0 < \lambda_\Omega < 1$. The second follows from $B_n \geq R_H$. $\square$

For constant relative position r and velocity v , a standard closest-approach calculation is

$$t_* = \text{clamp}_I \left(-\frac{r \cdot v}{\|v\|^2} \right), \quad d_{\min} = \|r + t_* v\|,$$

with the stationary case treated separately. In an exact finite implementation, squared distances and rational cross-products may be compared without introducing a square root.

A correct conservative theorem additionally requires:

- a declared dynamics model and coordinate frame;
- uncertainty bounds and conservative inflation rules;
- nested time windows that retain every time at which a true conjunction may occur;
- exact arithmetic or a fully specified directed-rounding policy;
- explicit decision predicates and versioned data.

Under those assumptions, the Omega hierarchy supplies deterministic refinement and the boundary witnesses supply replay. Performance and detection quality relative to established continuous-collision and conjunction-assessment methods remain empirical questions [16, 17].

15 Application III: scalar logical ordering

The 16-bit codebook can label finite seed positions with a reproducible increasing scalar. The distinction between scalar total ordering and causal partial ordering follows the classical logical-clock literature [13, 14, 15]. A scalar label is not, by itself, a causal clock. To define a total order across nodes, an explicit protocol may compare tuples such as

$$(\text{epoch}, T(b), \text{node identifier}, \text{local sequence}).$$

Lexicographic comparison gives a deterministic total order. This can support replay or canonical merge order, but it does not reveal concurrency in the manner of vector clocks. Seed assignment, reuse, epochs, and tie-breaking belong to the surrounding protocol rather than to the contraction kernel.

16 Application IV: golden-basis geometry and harmonic scaffold

Two secondary constructions from the source corpus are algebraically valid when kept distinct from the witness theorem.

16.1 Golden-basis automorphism

Represent $a + b\phi \in \mathbb{Z}[\phi]$ by the pair (a, b) . Multiplication by ϕ is

$$M_\phi = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \quad M_\phi^{-1} = \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}.$$

Both matrices have integer entries and determinant -1 , and

$$M_\phi M_\phi^{-1} = I.$$

Thus golden-basis up/down scaling is an exact automorphism of $\mathbb{Z}^2$. This operation is independent of the quotient-remainder contraction unless a larger protocol explicitly composes them.

16.2 Exponentially weighted harmonic series

For period $T > 0$, define

$$H_\infty(t) = \sum_{n=1}^{\infty} \phi^{-n} \sin\left(\frac{2\pi nt}{T}\right).$$

Absolute and uniform convergence follow from

$$\sum_{n=1}^{\infty} \phi^{-n} = \phi.$$

Hence H_∞ is continuous and T -periodic, with $|H_\infty(t)| \leq \phi$. Choosing $T = 23$ is a protocol convention; the bound is valid for every positive period.

17 Serialization, hashing, and integrity

A hash is meaningful only after the byte representation is fixed. RFC 8785 [18] specifies the JSON Canonicalization Scheme for the I-JSON subset, including deterministic property sorting and primitive serialization. Large exact integers that exceed the interoperable IEEE-754 integer range should be represented by a declared string profile or another unambiguous encoding.

The historical scripts use a stricter application profile: dictionaries, arrays, strings, integers, Booleans, and null; duplicate keys rejected; floating values, NaN, and infinity prohibited; keys sorted; compact separators; UTF-8 output. This is deterministic for the restricted data model but should not be called a complete RFC 8785 implementation unless its string and number serialization is independently verified against that RFC.

SHA-256 over a deterministic serialization provides a tamper-evident digest and implementation conformance check under the usual collision-resistance assumption. It does not authenticate origin without a keyed message-authentication code or digital signature. Algebraic witness reconstruction and cryptographic digest verification are different assurances and should be reported separately; this is also distinct from logical reversibility in the broader computational sense [6].

18 Reference implementation and verification

The companion file `omega_alpha_reference.py` uses only the Python standard library. It treats arbitrary-precision integer arithmetic as exact and uses `Decimal` only for verification-level evaluation of the symbolic constants. The finite protocol begins after S, Q, N are fixed.

The following exhaustive results were obtained:

The uploaded JPRV v0.2.4 strict implementation was also executed and reported

JPRV V0.2.4 strict reference self-test: PASS.

Table 2: Reference verification summary.

Test	Result
Protocol coefficient	$Q = 339949771344778$ rederived at 70 and 90 decimal digits
15-bit integer domain	All 32,768 values round-trip exactly; global identity holds
Terminal state	All 15-bit limbs are zero after 23 steps; maximum extinction depth is 10
15-bit byte folding	All declared test vectors round-trip exactly
Byte codebook	256 entries, strictly increasing, direct decoder exact
Byte codebook hash	0fc8922c2d84cfd965c9fe310070e00 ffd86ed76117584a9ff2faa85136a0c47
16-bit codebook	65,536 entries, strictly increasing, iterative candidate decoder plus replay exact
16-bit map hash	c0d90669759bfe4c2efc7692ba744962 49f9ff1f63748166afb19232724f8ff8
Negative tests	Altered ledgers, invalid padding, Boolean-as-integer, and non-codeword inputs rejected

19 Claim classification and limitations

The synthesis uses four labels.

Proved A statement derived from definitions and standard mathematics in this paper.

Exhaustively verified

A finite-domain claim checked for every input in the declared domain.

Convention A protocol choice that becomes exact once declared, but is not forced by mathematics.

Proposed application

A use that requires additional assumptions, implementation, or empirical evaluation.

The following are not established by the present results:

- that λ_Ω is universally optimal among contraction coefficients;
- that every geometric or physical system uniquely determines λ_Ω ;
- that finite witnesses recover pre-quantization information;
- that $N = 23$ is a universal maximum of reversibility;
- that the framework explains entropy, quantum measurement, dark matter, cosmology, or consciousness;
- that a value-plus-residual representation is compression without a storage-size analysis;
- that a deterministic codebook is cryptographically secure without a threat model;
- that an unkeyed hash authenticates an author or sender;
- that the collision-screening proposal outperforms established systems without benchmark evidence;
- that the ellipse coordinate replaces the complete elliptic integral.

These exclusions do not weaken the exact finite theorem. They identify the boundary between the proved construction and future research.

20 Discussion

The continuous and finite layers answer different questions. The affine map gives an exact scale law and a simple semigroup. The integer map gives a deterministic rational realization but loses information in its visible quotient. The ordered remainder ledger supplies precisely the information required for exact digital recovery. The multiscale probe then uses the scale law as a schedule and the witness system as an audit record.

The framework’s strongest architectural property is payload independence: the symbolic specification determines the scale coefficient before the data are known. That property is useful for reproducible protocols, but it is not unique to this coefficient; many computable constants are specification-derived. The distinctive choice here is the declared golden/circular normalization and the finite results obtained from its scaled floor.

The one-arcsecond construction should likewise be read as a fixed-aperture sampling convention. On a curved surface, the same tangent-space angle and Omega-scaled radii produce surface-dependent transverse measurements. In a computer, the same principle becomes a repeatable hierarchy of finite queries. The generality lies in the measurement interface, not in a claim that every target is literally a two-dimensional surface.

21 Conclusion

The Omega–Alpha framework consists of a precisely defined affine contraction and an exact finite witness lift. The continuous map contracts displacement toward a fixed point by the specification-derived coefficient $\phi/(\phi + \pi)$. The finite map replaces that coefficient by a declared scaled integer and stores every Euclidean remainder. The terminal state and ordered ledger recover the represented initial state exactly, and strict replay validates the path.

This combination supplies a deterministic multiscale probe: a scale generator, a measurement schedule, and an exact record of finite observations. The core mathematics is modest, explicit, and reproducible. Its usefulness in geometry, navigation, distributed ordering, or other domains depends on clearly stated measurement models and empirical evaluation rather than on broader metaphysical interpretations.

A Reference constants and test vectors

Quantity	Reference value
ϕ	1.61803398874989484820458683436563811772030917980576
π	3.14159265358979323846264338327950288419716939937511
λ_Ω	0.33994977134477892957670280932127919355100990501426
κ	1.07895740362416456641369720327285296513863541099576
λ_Ω^{23}	$1.66931896786205854202278982544429603856787514680458 \times 10^{-11}$
S	1,000,000,000,000,000
Q	339,949,771,344,778
N	23

B Toy witness example

To display the mechanism with small numbers, take $S' = 10000$, $Q' = 3400$, $N' = 4$, and $d_0 = 123$. The forward path is

$$123 \rightarrow 41 \rightarrow 13 \rightarrow 4 \rightarrow 1,$$

with remainders

$$(8200, 9400, 4200, 3600).$$

Reverse reconstruction gives

$$\begin{aligned} 4 &= \frac{10000 \cdot 1 + 3600}{3400}, & 13 &= \frac{10000 \cdot 4 + 4200}{3400}, \\ 41 &= \frac{10000 \cdot 13 + 9400}{3400}, & 123 &= \frac{10000 \cdot 41 + 8200}{3400}. \end{aligned}$$

This example is illustrative only; the reference profile uses the constants stated above.

C Minimal pseudocode

Listing 1: Core witnessed contraction and recovery.

```

1 def contract(d, S, Q, N):
2     remainders = []
3     for _ in range(N):
4         d_next, rho = divmod(Q * d, S)
5         remainders.append(rho)
6         d = d_next
7     return d, remainders
8
9
10 def reconstruct(d_final, remainders, S, Q):
11     d = d_final
12     for rho in reversed(remainders):
13         numerator = S * d + rho
14         d, remainder = divmod(numerator, Q)
15         if remainder != 0:
16             raise ValueError("invalid witness")
17     return d

```

D Corpus correction summary

The master synthesis applies the following principal corrections to the historical archive:

1. Decimal approximations of λ_Ω and κ were corrected.
2. Symbolic exactness, finite decimal evaluation, and exact integer arithmetic were separated.
3. Unsupported claims of optimal irrationality and rational-loop prevention were removed.
4. The 23-step value was recast as a fixed protocol horizon, with any guard-band derivation made conditional on an explicit threshold.
5. The floor map was identified as many-to-one; exactness was assigned to the augmented ordered witness.
6. “Involution” was replaced by “injective encoding with a left inverse on valid witnesses.”
7. Monotone codebooks were separated from physical clocks and from causal-order reconstruction.
8. The ellipse application was narrowed to a finite coordinate for a selected multiplier, and inconsistent table entries were corrected.
9. Physical interpretations involving entropy, quantum mechanics, cosmology, the dark sector, and consciousness were excluded.

10. Hashing was described as integrity binding, not authentication; residual storage was not called compression without a size model.

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