
OMEGA Marked Reference Geometry

A One-Arcsecond Seed, Center-Relative Reconstruction,
and Positive-Homothety Scaling

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Paper VII in the OMEGA / OMEGA Naught sequence. This paper formalizes the local marked Euclidean kernel, its marked measurement spine, center-relative scale transport, and compatible dimensional reconstruction criterion. It does not claim that the exploratory outer Echo-LoCo completion is determined by the one-arcsecond seed.

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Abstract

This paper gives an exact Euclidean definition of the local OMEGA marked reference geometry suggested by the author's one-arcsecond construction. The seed consists of an oriented unit circle, a distinguished center, a unit bisecting direction, an ordered pair of radius endpoints with central separation exactly one arcsecond, and the distinguished minor arc between them. The chord, minor-arc length, and sagitta then follow uniquely. Declared collinear extensions of lengths 1 and ϕ produce the total extension $\phi^2 = 1 + \phi$ on each side. The resulting marked local kernel is unique up to orientation-preserving Euclidean isometry. The theorem is explicitly separated from the still-open specification of the larger hand-drawn outer completion.

Positive uniform scaling produces a one-parameter family of similar marked objects. The multiplicative group $(\mathbb{R}_{>0}, \times)$ acts freely and transitively on the canonically positioned family, so the family is a torsor for positive homotheties about the distinguished center. A finite marked configuration is stored exactly by that center, a positive scale, normalized center-relative vectors, and sufficient orientation data. Its configuration Gram matrix scales by the square of the common scale, while normalized Gram and distance data are invariant. The constants $\Omega_0 = \phi + \pi$, $\lambda = \phi/\Omega_0$, and $\mu = \pi/\Omega_0$ define a complementary one-dimensional marked measurement spine. The planar kernel and spine are retained as distinct typed structures and may be paired under a diagonal scaling action. Compatible isometric embeddings preserve marked subobjects across dimensions. Lower- and higher-dimensional constraints may reconstruct an intermediate realization only when their common admissible set is proved unique, including orientation and branch choices. An exact external interface to the frozen Scale-Resolved State Dynamics encoder is also recorded. No physical dynamics, automatic dimensional recovery, universal computational privilege, or new Euclidean primitive is asserted.

The relationship to Papers I–VI is made explicit through typed maps rather than a claim of common derivation. The rational floor/remainder step is an exact instance of the frozen SRSD encoder, the OMEGA arithmetic gives an exact coordinate bridge for the quadratic field used by the cubature branch, and positive homothety transports the spherical sampling and cubature rules across radii.

Keywords. marked Euclidean geometry; arcsecond; golden ratio; homothety; center-relative coordinates; Gram matrix; compatible embedding; constraint reconstruction; torsor; similarity invariant; exact replay; scale-resolved encoding; icosahedral cubature

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1 Purpose, vocabulary, and claim boundary

The purpose of this paper is to turn a geometric sketch into an auditable mathematical object. The decisive local angular input is exactly one arcsecond on a unit circle. Once an orientation, a bisecting direction, an ordering of the two radius endpoints, and the stated golden-ratio extensions are fixed, the local configuration has no remaining geometric freedom. That is the theorem established here.

The local theorem must not be confused with a stronger statement about every arc, tangent, rectangle, vertical return, or named point in the author’s exploratory notebooks. Those features become mathematical consequences only after their incidence and metric rules are stated as equations or axioms. Accordingly:

- *local OMEGA kernel* means the exact circle, angular seed, distinguished minor arc, chord, and golden extensions defined in Sections 3–4;
- *OMEGA marked scaling family* means the positive scale copies of the planar kernel or of the separately defined measurement spine;
- *center-relative reconstruction* means exact recovery of marks whose normalized vectors and required frame data have already been specified; it does not infer coordinates from a drawing;
- *outer completion* means the larger Echo–Loco style diagram, whose complete rule set is not assumed here.

Naming and vocabulary. OMEGA, OMEGA Naught, Echo, and Loco are project-specific names for author-defined constructions and artifacts. They do not denote new standard mathematical primitives and do not confer mathematical privilege on the standard operations used below. Names locate the objects; the printed definitions govern their meaning.

Claim boundary 1.1 (Principal statement) *The local OMEGA reference kernel is uniquely determined, up to orientation-preserving Euclidean isometry, by its declared rule set. The complete outer OMEGA figure is not claimed to be uniquely determined until all of its completion rules have been declared. The center-relative and dimensional reconstruction results transport or intersect exact declared data; they do not supply missing outer constraints.*

This distinction identifies exactly what is proved, which quantities are derived, and where a future specification must add information.

The paper also records exact interfaces to Papers I–VI. An interface means a typed identity, specialization, or transport map. It does not mean that the connected objects have the same type, that one paper retrospectively proves another, or that the one-arcsecond seed generates the entire sequence.

2 Constants, units, and conventions

2.1 OMEGA constants

Let

$$\phi = \frac{1 + \sqrt{5}}{2}, \quad \Omega_0 = \phi + \pi, \quad \tau = \frac{1}{\Omega_0}, \quad \lambda = \frac{\phi}{\Omega_0}, \quad \mu = \frac{\pi}{\Omega_0}. \quad (1)$$

Then

$$\phi^2 = \phi + 1, \quad \lambda + \mu = 1, \quad \lambda^2 - \tau\lambda - \tau^2 = 0, \quad 0 < \lambda, \mu < 1. \quad (2)$$

These are exact real constants. Decimal values are aids to verification, not parts of the definitions. The identity $\phi^2 = 1 + \phi$ governs the local collinear extensions. The sum $\Omega_0 = \phi + \pi$ governs the marked one-dimensional measurement spine in Section 7. Neither identity selects a dynamical law. The relation $\lambda^2 - \tau\lambda - \tau^2 = 0$ records the exact arithmetic coordinate used later to connect Paper V with the quadratic-field coefficients of Paper VI.

2.2 One arcsecond

The angular seed is

$$\varepsilon = 1'' = \frac{1}{3600}^\circ = \frac{\pi}{648000} \text{ radians}. \quad (3)$$

On a circle of radius a , the corresponding minor-arc length is $a\varepsilon$. The angle is dimensionless; the arc length depends on the chosen radius.

2.3 Orientation and marked data

We work in an oriented Euclidean plane. A marked object includes its distinguished points, curves, directions, labels, and roles. An isometry of a marked object must preserve this typed data and the endpoint ordering, not merely the unlabelled point set. For nonzero vectors v, w , write $\text{ang}^+(v, w) \in [0, 2\pi)$ for the positively oriented angle from v to w , and write R_θ for positively oriented rotation through θ .

3 The one-arcsecond Euclidean seed

Definition 3.1 (Oriented one-arcsecond seed) *An oriented one-arcsecond seed is the marked tuple*

$$\mathcal{S}_\varepsilon = (E^2, \text{or}, \alpha, C, u; d, b; A_\varepsilon),$$

where E^2 is an oriented Euclidean plane, $\alpha \in E^2$, $C = \{x : \|x - \alpha\| = 1\}$, and u is a unit direction based at α . Its ordered endpoints and distinguished minor arc are

$$d = \alpha + R_{-\varepsilon/2}u, \quad b = \alpha + R_{\varepsilon/2}u, \quad (4)$$

$$A_\varepsilon = \{\alpha + R_\theta u : -\varepsilon/2 \leq \theta \leq \varepsilon/2\}. \quad (5)$$

Thus $\text{ang}^+(d - \alpha, b - \alpha) = \varepsilon$, and u is the internal bisector of the distinguished minor arc.

The labels d and b record left and right endpoints in the canonical placement below. Exchanging them changes the oriented marked object. If an open-circle rendering is desired while retaining the endpoint marks, define the endpoint-free distinguished arc and its complement by

$$A_\varepsilon^\circ = \{\alpha + R_\theta u : -\varepsilon/2 < \theta < \varepsilon/2\}, \quad C^\circ = C \setminus A_\varepsilon^\circ. \quad (6)$$

The kernel itself records A_ε as a distinguished arc; it does not silently delete it.

Proposition 3.2 (Canonical coordinates) *Every oriented one-arcsecond seed has an orientation-preserving Euclidean placement with*

$$\alpha = (0, 0), \quad u = (0, -1), \quad (7)$$

and

$$d = \left(-\sin \frac{\varepsilon}{2}, -\cos \frac{\varepsilon}{2}\right), \quad b = \left(\sin \frac{\varepsilon}{2}, -\cos \frac{\varepsilon}{2}\right). \quad (8)$$

The chord $[d, b]$ is horizontal.

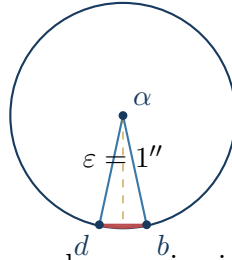
Proof. Translate α to the origin and rotate u to the negative y -axis. Equation (4) then gives the displayed coordinates. The two y -coordinates agree. $\square$

Proposition 3.3 (Forced local measurements) *For the unit-radius seed, the distinguished minor-arc length, chord length, and sagitta are*

$$L_{\text{arc}} = \varepsilon, \quad L_{\text{chord}} = 2 \sin \frac{\varepsilon}{2}, \quad h = 1 - \cos \frac{\varepsilon}{2} = 2 \sin^2 \frac{\varepsilon}{4}. \quad (9)$$

The complementary circular arc has length $2\pi - \varepsilon$.

Proof. The unit-circle arc-length formula gives $L_{\text{arc}} = \varepsilon$. Bisecting the isosceles triangle subtended by the chord gives half-chord $\sin(\varepsilon/2)$ and apothem $\cos(\varepsilon/2)$. The stable sagitta identity is $1 - \cos x = 2 \sin^2(x/2)$. $\square$



Schematic only: the angular opening is greatly exaggerated.
Exact chord length: $2 \sin(\varepsilon/2)$.

Figure 1: The exact local seed. The red curve is the distinguished minor arc A_ε ; an open-circle rendering may remove A_ε° .

The exact chord length is not equal to the radian measure ε , although the two are extremely close:

$$2 \sin \frac{\varepsilon}{2} = \varepsilon - \frac{\varepsilon^3}{24} + O(\varepsilon^5). \quad (10)$$

Thus “one arcsecond apart horizontally” means that the endpoints have one arcsecond of central angular separation and their connecting chord is horizontal in the canonical placement.

Translating the chord line to $y = 0$ gives

$$d = \left(-\sin \frac{\varepsilon}{2}, 0\right), \quad b = \left(\sin \frac{\varepsilon}{2}, 0\right), \quad \alpha = \left(0, \cos \frac{\varepsilon}{2}\right). \quad (11)$$

4 Golden-ratio extensions

Let

$$e = \frac{b - d}{\|b - d\|} \quad (12)$$

be the positively directed unit vector along the chord. In Equation (11), $e = (1, 0)$.

Definition 4.1 (Declared bilateral extensions) *Define*

$$d_1 = d - e, \quad d_2 = d - \phi^2 e, \quad (13)$$

$$b_1 = b + e, \quad b_2 = b + \phi^2 e. \quad (14)$$

The directions are outward from the seed chord, and the points occur in the order d_2, d_1, d, b, b_1, b_2 along the oriented baseline.

Proposition 4.2 (Golden decomposition) *The bilateral extensions satisfy*

$$\|d - d_1\| = \|b - b_1\| = 1, \quad (15)$$

$$\|d_1 - d_2\| = \|b_1 - b_2\| = \phi, \quad (16)$$

$$\|d - d_2\| = \|b - b_2\| = \phi^2. \quad (17)$$

Proof. The first and third equalities follow from $\|e\| = 1$. For the intervening segment,

$$\|d_1 - d_2\| = \|(-e) - (-\phi^2 e)\| = \phi^2 - 1 = \phi,$$

and the right-hand calculation is identical. $\square$

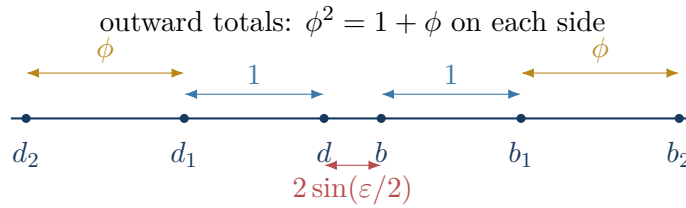


Figure 2: The declared collinear golden extensions. Horizontal proportions are schematic; the central chord is too small to resolve at this scale.

The golden extensions are not consequences of the one-arcsecond angle. They are additional declared rules. Once declared, their positions are forced.

5 The local OMEGA marked kernel

Definition 5.1 (Local OMEGA reference kernel) *The unit-scale local OMEGA reference kernel is*

$$\mathcal{K}_\Omega(1) = (E^2, \text{or}, \alpha, C, u; d, b; A_\epsilon; d_1, d_2, b_1, b_2), \quad (18)$$

subject to the definitions above.

Theorem 5.2 (Local rigidity) *Any two realizations of $\mathcal{K}_\Omega(1)$ are related by a unique orientation-preserving Euclidean isometry preserving every named mark and its type. Equivalently, the local OMEGA reference kernel is uniquely determined up to orientation-preserving Euclidean isometry.*

Proof. Given two realizations, there is a unique direct Euclidean isometry sending α to α' and u to u' : translate the center and apply the unique orientation-preserving rotation matching the unit directions. Equation (4) forces the images of d, b , and Equation (5) forces the image of A_ε . The directed chord vector e is therefore sent to e' , so Equations (13)–(14) force the images of all extension marks. If two direct isometries had these properties, their quotient would fix a point and a nonzero directed vector and hence be the identity. $\square$

Corollary 5.3 (Predetermined local shape) *Once the endpoint ordering, unit radius, bisecting unit direction, plane orientation, and golden-extension rules are fixed, every distance and incidence relation among the marks of $\mathcal{K}_\Omega(1)$ is predetermined.*

The precise motivating statement is therefore:

The one-arcsecond angle fixes the circular seed up to similarity; unit radius fixes its direct-isometry class. The additional bilateral $(1, \phi)$ extension rule fixes the local marked kernel.

No unlisted outer arc, box edge, or named outer point follows from the angle alone.

6 The outer-completion boundary

The author’s sketches contain larger circles or arcs, vertical lines, rectangular boundaries, an inner marked region, and points named Echo and Loco. A drawing can motivate a construction, but visual placement is not an exact incidence axiom. Table 1 records the present logical status.

Definition 6.1 (Outer completion data) *An outer completion rule set $\mathcal{R}_{\text{out}}$ is a finite or locally finite typed family of additional points, curves, orderings, incidence relations, distance equations, angle equations, and tangency conditions, together with a distinguished center, declared ambient realization spaces, dimensional memberships, orientation and branch data, and an equivalence group, containing $\mathcal{K}_\Omega(1)$ as a marked subobject.*

For any proposed $\mathcal{R}_{\text{out}}$, one must:

1. prove existence of a Euclidean realization;
2. determine uniqueness up to the declared equivalence group;
3. identify redundant constraints and discrete branch or mirror choices;
4. provide exact coordinates or certified interval/algebraic reconstruction.

Until then, the notebook figure proposes an outer completion; the angle alone does not prove it. In particular, drawn placement does not define coordinates for Space, Echo, or Loco.

Table 1: Logical status of the local and outer components.

Component	Present status	Requirement for a uniqueness theorem
One-arcsecond circle seed	Exact and proved	No additional rule
Golden baseline marks	Exact once declared	No additional rule
Positive scale copies	Exact after typed scaling	No additional rule
Auxiliary circles	Exploratory	Centers, radii, and intended intersections or tangencies
Curved side returns	Exploratory	Arc centers, endpoints, orientation, and tangency or curvature conditions
Echo-Loce frame	Exploratory	Exact center-relative coordinates or constraints for every edge and named point
“Space” diamond	Interpretive; not yet typed as a point	Ambient dimension, exact relations, branch data, and existence and uniqueness certificates
“angular alpha”	Interpretive label	Definition of the marked angle or region and its mathematical role
Analytic OMEGA operators	Separate branch	Domain, codomain, parameters, and a theorem connecting the operator to this geometry

7 The OMEGA marked measurement spine

The constants ϕ and π define a separate one-dimensional marked object useful for normalization and scale bookkeeping.

Definition 7.1 (Marked measurement spine) *For $a > 0$, let*

$$\Gamma_{\Omega}(a) = ([0, a\Omega_0], d_E; p_0(a), p_{\phi}(a), p_{\Omega}(a)), \quad (19)$$

where $d_E(x, y) = |x - y|$ and

$$p_0(a) = 0, \quad p_{\phi}(a) = a\phi, \quad p_{\Omega}(a) = a\Omega_0. \quad (20)$$

The consecutive segment lengths are $a\phi$ and $a\pi$. The number $a\pi$ is a length, not an additional marked point. Normalizing by total length with $N_a(x) = x/(a\Omega_0)$ gives

$$\Gamma_{\Omega} = ([0, 1], d_E; 0, \lambda, 1), \quad 1 - \lambda = \mu. \quad (21)$$

Thus λ, μ are the two normalized subinterval proportions; only λ is the interior marked coordinate.

Remark 7.2 (Relation to the planar kernel) *The spine and planar kernel share ϕ and the positive scale parameter a , but they are different typed structures. A measurement, projection, or path-length map must be declared before a planar feature can be identified with a spine coordinate.*

8 Positive-homothety scaling

8.1 Center-relative normalized configurations

Let $\alpha \in \mathbb{R}^n$ be a distinguished center. For each marked point p_i , its primary center-relative vector is

$$r_i = p_i - \alpha. \quad (22)$$

If $a > 0$ is the common scale, define the normalized vector

$$\xi_i = \frac{p_i - \alpha}{a}. \quad (23)$$

Thus absolute coordinates are recovered from center, scale, and normalized data by

$$p_i(a) = \alpha + a\xi_i. \quad (24)$$

Theorem 8.1 (OMEGA center-relative reconstruction principle) *Let $\Xi = (\xi_1, \dots, \xi_N) \subset \mathbb{R}^n$ be an exactly specified ordered finite marked configuration relative to the vector-space origin. For every $\alpha \in \mathbb{R}^n$ and $a > 0$, Equation (24) determines a unique ordered marked realization. For $c > 0$, the homothety about α ,*

$$H_{\alpha,c}(x) = \alpha + c(x - \alpha), \quad (25)$$

satisfies

$$p_i(ca) = H_{\alpha,c}(p_i(a)), \quad p_i(ca) - \alpha = c(p_i(a) - \alpha). \quad (26)$$

Consequently, differences, lengths, squared lengths, inner products, and Euclidean k -volumes have homogeneous degrees 1, 1, 2, 2, k , respectively. Angles, normalized inner products, scale-free ratios, and all marked incidence and orientation relations preserved by a positive similarity have degree zero.

Proof. The reconstruction formula gives each point explicitly, so uniqueness is immediate. Equation (26) follows by substitution. Every remaining statement follows from multilinearity of inner products and determinants and cancellation of common positive factors in normalized quantities. $\square$

For the complete configuration, collect the normalized and center-relative vectors as columns:

$$X = [\xi_1 \ \cdots \ \xi_N], \quad R(a) = [p_1(a) - \alpha \ \cdots \ p_N(a) - \alpha] = aX. \quad (27)$$

Its *configuration Gram matrix* is

$$\text{Gr}(a) = R(a)^T R(a) = a^2 X^T X, \quad \widehat{\text{Gr}} = \frac{1}{a^2} \text{Gr}(a) = X^T X. \quad (28)$$

Thus $\text{Gr}(ca) = c^2 \text{Gr}(a)$, while the normalized Gram matrix is scale-independent. Its entries give center distances and pairwise distances:

$$\text{Gr}_{ii}(a) = \|p_i(a) - \alpha\|^2, \quad (29)$$

$$\|p_i(a) - p_j(a)\|^2 = \text{Gr}_{ii}(a) + \text{Gr}_{jj}(a) - 2\text{Gr}_{ij}(a). \quad (30)$$

Equivalently, the normalized distance matrix

$$\widehat{D}_{ij} = \frac{\|p_i(a) - p_j(a)\|}{a} = \|\xi_i - \xi_j\| \quad (31)$$

does not depend on a .

Remark 8.2 (Metric data and oriented realization) *The normalized Gram matrix determines a center-relative Euclidean configuration only up to an orthogonal transformation on its span. It does not select an absolute ambient placement or distinguish a realization from its reflection. A specific oriented realization therefore also requires sufficient anchor or frame data; reflection and other discrete branches must not be discarded silently.*

One convenient exact record for a completely specified realization is

$$\mathfrak{D} = (\alpha, a, E, \Xi, \mathcal{C}, \mathcal{M}), \quad (32)$$

where E is an oriented ambient frame, $\mathcal{C}$ is an exact typed constraint ledger, Ξ is expressed relative to E , and $\mathcal{M}$ records mark types, labels, incidence roles, and dimensional memberships. Under T_c , only a and the dimensionful entries of $\mathcal{C}$ acquire their prescribed powers of c ; $E, \Xi, \mathcal{M}$, and the scale-free relations remain fixed. This tuple is bookkeeping for standard Euclidean data, not a new geometric primitive.

In compact form,

$$\boxed{\text{center} + \text{relations} + \text{frame} + \text{scale} \implies \text{reconstruction}}, \quad (33)$$

$$\boxed{\text{scale state changes; normalized relational geometry does not}}.$$

8.2 Scaled planar kernels and the paired object

The local kernel is the special case with canonical center $\alpha = 0$. For $a > 0$, define the scaled data componentwise by

$$C_a = aC, \quad A_{\varepsilon, a} = aA_{\varepsilon}, \quad d_a = ad, \quad b_a = ab, \quad (d_{1,a}, d_{2,a}, b_{1,a}, b_{2,a}) = a(d_1, d_2, b_1, b_2), \quad (34)$$

while the marked unit direction is

$$u_a = u. \quad (35)$$

Let $\mathcal{K}_{\Omega}(a)$ be the resulting typed marked kernel. Its circle has radius a , its central angle remains ε , and its extension lengths are $a, a\phi, a\phi^2$.

For $c > 0$, the origin-centered homothety $H_c(x) = cx$ acts on the point and curve data and induces the unchanged normalized direction u . Define

$$T_c(\mathcal{K}_{\Omega}(a)) = \mathcal{K}_{\Omega}(ca). \quad (36)$$

Theorem 8.3 (Scaling action) *The map*

$$\rho : \mathbb{R}_{>0} \times \{\mathcal{K}_{\Omega}(a) : a > 0\} \longrightarrow \{\mathcal{K}_{\Omega}(a) : a > 0\}, \quad \rho(c, \mathcal{K}_{\Omega}(a)) = \mathcal{K}_{\Omega}(ca), \quad (37)$$

is a free and transitive action of $(\mathbb{R}_{>0}, \times)$.

Proof. The identity and composition laws are

$$T_1(\mathcal{K}_{\Omega}(a)) = \mathcal{K}_{\Omega}(a), \quad T_b(T_c(\mathcal{K}_{\Omega}(a))) = \mathcal{K}_{\Omega}(bca) = T_{bc}(\mathcal{K}_{\Omega}(a)).$$

Also $T_c^{-1} = T_{1/c}$. If $T_c(\mathcal{K}_{\Omega}(a)) = \mathcal{K}_{\Omega}(a)$, equality of circle radii gives $ca = a$, hence $c = 1$. Given $\mathcal{K}_{\Omega}(a)$ and $\mathcal{K}_{\Omega}(b)$, the unique $c = b/a$ sends the first to the second. $\square$

The same proof applies to the spine family

$$\mathcal{F}_\Omega = \{\Gamma_\Omega(a) : a > 0\}, \quad T_c(\Gamma_\Omega(a)) = \Gamma_\Omega(ca). \quad (38)$$

Each canonically positioned family is a torsor for $(\mathbb{R}_{>0}, \times)$.

To package both structures without identifying them, define

$$\mathfrak{G}_\Omega(a) = (\mathcal{K}_\Omega(a), \Gamma_\Omega(a)), \quad T_c \mathfrak{G}_\Omega(a) = \mathfrak{G}_\Omega(ca). \quad (39)$$

This is a diagonal action on a typed pair, not an assertion that the planar kernel and marked interval are the same object.

8.3 Scaling laws

A scalar quantity Q is homogeneous of degree k if

$$Q(T_c X) = c^k Q(X). \quad (40)$$

Table 2: Scale-dependent quantities and invariants.

Quantity at scale a	Under $a \mapsto ca$	Degree/status
Circle radius a	multiplied by c	1
Distinguished minor-arc length $a\varepsilon$	multiplied by c	1
Chord $2a \sin(\varepsilon/2)$	multiplied by c	1
Sagitta $a(1 - \cos(\varepsilon/2))$	multiplied by c	1
Extension lengths $a, a\phi, a\phi^2$	multiplied by c	1
Squared distance and planar area	multiplied by c^2	2
Configuration Gram entries	multiplied by c^2	2
Angle ε and unit direction u	unchanged	0
Ratios $1 : \phi : \phi^2$	unchanged	0
Normalized distances and normalized Gram data	unchanged	0
Spine subinterval proportions λ, μ	unchanged	0
Labels, order, and incidence	preserved for $c > 0$	invariant

The family is deliberately indexed by $a > 0$. At $c = 0$ the configuration collapses. In the plane, $c < 0$ gives $H_c = R_\pi \circ H_{|c|}$: it preserves ambient plane orientation but takes the object out of the chosen canonical placement. On the one-dimensional spine it also reverses ambient order. Thus the acting group is $(\mathbb{R}_{>0}, \times)$.

8.4 Action versus dynamics

The action specifies available transformations; it does not select which transformation occurs. A separate scalar dynamics may choose

$$a_{k+1} = c_k a_k, \quad c_k > 0. \quad (41)$$

Then

$$a_n = a_0 \prod_{k=0}^{n-1} c_k, \quad \mathcal{K}_\Omega(a_n) = T_{C_n} \mathcal{K}_\Omega(a_0), \quad C_n = \prod_{k < n} c_k. \quad (42)$$

The ordered history contains intermediate-path information that the final product does not retain. With $v = \log a$, multiplication becomes addition:

$$v_{k+1} = v_k + \log c_k, \quad \log \frac{a_n}{a_0} = \sum_{k < n} \log c_k. \quad (43)$$

This is the standard isomorphism $(\mathbb{R}_{>0}, \times) \cong (\mathbb{R}, +)$, not an additional OMEGA operation.

9 Compatible dimensional realizations and conditional reconstruction

Increasing ambient dimension does not make distinct objects identical and does not create missing information. Compatibility means that explicit maps preserve the metrics, named marks, and relations claimed to survive.

9.1 Axis-marked metric realizations

Let $G \in \text{Sym}_n(\mathbb{R})$ be positive definite, meaning $x^T G x > 0$ for every $x \neq 0$, and define

$$\langle x, y \rangle_G = x^T G y, \quad d_G(x, y) = \sqrt{(x - y)^T G (x - y)}. \quad (44)$$

Here G is the matrix of the *ambient metric*; it is not the configuration Gram matrix Gr in Equation (28). An axis-marked cubical realization is

$$P_{\Omega, n}(a) = [0, a\Omega_0]^n \quad (45)$$

with marks

$$o = 0, \quad p_{\phi, i} = a\phi e_i, \quad p_{\Omega, i} = a\Omega_0 e_i \quad (1 \leq i \leq n). \quad (46)$$

Write the marked metric object as

$$\Gamma_{\Omega, n}(a; G) = (P_{\Omega, n}(a), d_G; o, (p_{\phi, i})_{i=1}^n, (p_{\Omega, i})_{i=1}^n). \quad (47)$$

On axis i , the marked distances are

$$d_G(o, p_{\phi, i}) = a\phi\sqrt{G_{ii}}, \quad d_G(o, p_{\Omega, i}) = a\Omega_0\sqrt{G_{ii}}. \quad (48)$$

Hence the normalized interior proportion is always λ , while the effective scale on axis i is $a\sqrt{G_{ii}}$. To retain the same scale a on every coordinate axis, require $G_{ii} = 1$ for all i , or use a G -orthonormal basis.

Uniform scaling about the center gives

$$d_G(H_{\alpha, c}x, H_{\alpha, c}y) = c d_G(x, y), \quad \text{Vol}_G(P_{\Omega, n}(ca)) = c^n \text{Vol}_G(P_{\Omega, n}(a)), \quad (49)$$

where, in the canonically centered cubical realization,

$$\text{Vol}_G(P_{\Omega, n}(a)) = (a\Omega_0)^n \sqrt{\det G}. \quad (50)$$

Angles are invariant because the common positive factors cancel in the cosine formula. The homothety law is dimension-independent; what changes with dimension is the frame and constraint information needed to identify a realization.

9.2 Compatible embeddings and marked subobjects

Let $(\mathbb{R}^m, G_m, \alpha_m)$ and $(\mathbb{R}^n, G_n, \alpha_n)$, with $m < n$, be centered Euclidean realizations. Let $J_m^n \in \mathbb{R}^{n \times m}$ have rank m , and define the affine embedding

$$\iota_m^n(x) = \alpha_n + J_m^n(x - \alpha_m). \quad (51)$$

It is isometric precisely when

$$(J_m^n)^T G_n J_m^n = G_m. \quad (52)$$

This is a pullback identity for the *ambient metric matrices*. If X_m contains normalized vectors for common named marks and $X_n = J_m^n X_m$, then their configuration Gram matrices agree:

$$X_n^T G_n X_n = X_m^T G_m X_m. \quad (53)$$

The embedding also commutes with center-relative scaling:

$$\iota_m^n \circ H_{\alpha_m, c} = H_{\alpha_n, c} \circ \iota_m^n. \quad (54)$$

For the canonical chain, one may take $J_n^{n+1}x = (x, 0)$. Then

$$\iota_n^{n+1}(P_{\Omega, n}(a)) = P_{\Omega, n+1}(a) \cap \{x_{n+1} = 0\}, \quad (55)$$

and the common named marks are preserved. More generally, a compatible chain

$$\mathcal{X}_2 \xrightarrow{\iota_2^3} \mathcal{X}_3 \xrightarrow{\iota_3^4} \mathcal{X}_4 \quad (56)$$

requires every common named mark and relation to commute with the displayed embeddings. This is the precise sense in which a lower-dimensional realization survives as a marked subobject.

A projection is not an embedding. A map from four dimensions to three may discard a coordinate or identify distinct points. Exact recovery from such a projection requires additional data that separate every point in the relevant fiber. Likewise, a three-dimensional embedded subobject, an affine section, and a projected image are different typed constructions.

9.3 Frames generated from the seed

In an oriented standard Euclidean three-space, let

$$r_d = d - \alpha, \quad r_b = b - \alpha, \quad \|r_d\| = \|r_b\| = a, \quad \angle(r_d, r_b) = \varepsilon. \quad (57)$$

Because $0 < \varepsilon < \pi$, the vectors are noncollinear and their internal bisector and ordered unit normal are

$$u = \frac{r_d/\|r_d\| + r_b/\|r_b\|}{\|r_d/\|r_d\| + r_b/\|r_b\|\|}, \quad n = \frac{r_d \times r_b}{\|r_d \times r_b\|}. \quad (58)$$

With $v = n \times u$, the ordered triple (u, v, n) , together with α , is an oriented orthonormal local frame. The order (d, b) and the ambient orientation are part of the data; reversing either can change the normal. Positive homothety leaves this normalized frame unchanged.

In $\mathbb{R}^n$, an ordered independent frame $E = (e_1, \dots, e_n)$ gives unique coordinates

$$p_i - \alpha = \sum_{k=1}^n x_{ik} e_k. \quad (59)$$

For an orthonormal frame, these are the usual inner-product coordinates. In four dimensions there is no ordinary vector-valued cross product with the role used in Equation (58); orientation is recorded by an ordered four-frame, for example through

$$\text{sgn det}[e_1 \ e_2 \ e_3 \ e_4]. \quad (60)$$

9.4 Constraint-intersection reconstruction

Suppose an unknown ordered tuple $x = (x_1, \dots, x_M) \in (\mathbb{R}^3)^M$ must obey exact constraints inherited from a declared two-dimensional marked subobject, constraints pulled back from a compatible four-dimensional reference, and intrinsic three-dimensional incidence, orientation, and branch conditions. After expressing all constraints in the same three-dimensional coordinates, write its admissible set as

$$\mathcal{A} = \{x : F_2(x) = 0, F_3(x) = 0, F_4(x) = 0, B(x)\}, \quad (61)$$

where B collects inequalities, orientation signs, and discrete branch choices.

Proposition 9.1 (Intermediate-dimensional reconstruction criterion) *The tuple is exactly reconstructed when $\mathcal{A}$ is a singleton. More generally, relative to a declared equivalence group $\mathcal{E}$, it is unique up to equivalence exactly when $\mathcal{A}$ is one $\mathcal{E}$ -orbit. Compatible lower- and higher-dimensional data do not imply either conclusion unless existence and this uniqueness condition are proved.*

This criterion may be summarized by the typed constraint pattern

$$\mathcal{X}_2 \longrightarrow \mathcal{X}_3 \longleftarrow \mathcal{X}_4, \quad \mathcal{A}/\mathcal{E} = \{[x_*]\}, \quad (62)$$

where the arrows denote declared constraint transport, not automatic recovery or generation of the intermediate object.

For one unknown point, typical exact equations include

$$\|x - a_i\|^2 = r_i^2, \quad (x - \alpha) \cdot v_j = c_j, \quad (x - \alpha) \cdot n = h. \quad (63)$$

Three generic sphere equations in $\mathbb{R}^3$ commonly leave zero, one, or two solutions; when two remain, they are mirror branches across the anchor plane. A further independent condition can select one branch.

For a smooth equality-constraint map $F : (\mathbb{R}^3)^M \rightarrow \mathbb{R}^r$, after the allowed Euclidean gauge freedoms are fixed, full column rank of $DF(x_*)$ is a sufficient test that x_* is locally isolated. It does not prove global uniqueness: disconnected or reflected solutions must still be excluded. Accordingly,

$$\text{existence} \neq \text{local rigidity} \neq \text{global rigidity}. \quad (64)$$

The same constraint discipline applies when a three-dimensional realization is a declared affine subspace $H \subset \mathbb{R}^4$. A point belongs to that realization only if it lies in H , and its three-dimensional coordinates require a declared affine isometry $\mathbb{R}^3 \rightarrow H$. A fourth coordinate is mathematical data, not automatically time, energy, spacetime, or another physical quantity.

Exact symbolic inputs may support exact reconstruction. Certified interval or measured approximate inputs should instead produce certified regions or uncertainty sets unless additional analysis proves an exact conclusion. The reference geometry does not turn insufficient or noisy observations into unique exact coordinates.

This section supplies compatible families and a reconstruction criterion. It does not assert that every higher-dimensional geometry carries a canonical OMEGA measurement map, or that the current notebook marks already supply the required constraint system.

10 External interface to frozen SRSD

Paper III fixes Scale-Resolved State Dynamics (SRSD) as a separate coordinate representation system [3]. The present geometry does not alter that specification. OMEGA scaling is external application data placed around the frozen encoder and decoder.

Let $a > 0$ be the geometric scale state and $s > 0$ an SRSD resolution:

$$a = \text{geometric state}, \quad s = \text{coordinate resolution}. \quad (65)$$

For each $s > 0$, define the frozen scalar maps

$$E_s : \mathbb{R} \longrightarrow \mathbb{Z} \times [0, s), \quad E_s(y) = (q_s(y), r_s(y)) = \left(\left\lfloor \frac{y}{s} \right\rfloor, y - s \left\lfloor \frac{y}{s} \right\rfloor \right), \quad (66)$$

$$D_s : \mathbb{Z} \times [0, s) \longrightarrow \mathbb{R}, \quad D_s(d, \rho) = sd + \rho. \quad (67)$$

They are mutually inverse. Restricted to the positive observable $y = a > 0$, the encoder's image is

$$\mathcal{E}_s^+ = \{(d, \rho) \in \mathbb{Z} \times [0, s) : sd + \rho > 0\}. \quad (68)$$

Use the radius observable

$$\text{rad}(\mathcal{K}_\Omega(a)) = a. \quad (69)$$

Suppose the geometric state undergoes $M_c(y) = cy$, $c > 0$, while the SRSD resolution changes from s to t . The exact resolved-coordinate transport is

$$\hat{T}_{c;s \rightarrow t} = E_t \circ M_c \circ D_s. \quad (70)$$

For $e = (d, \rho)$,

$$d' = \left\lfloor \frac{c(sd + \rho)}{t} \right\rfloor, \quad (71)$$

$$\rho' = c(sd + \rho) - t \left\lfloor \frac{c(sd + \rho)}{t} \right\rfloor. \quad (72)$$

The inverse and composition laws are

$$\hat{T}_{c;s \rightarrow t}^{-1} = E_s \circ M_{1/c} \circ D_t, \quad (73)$$

$$\hat{T}_{b;t \rightarrow u} \circ \hat{T}_{c;s \rightarrow t} = \hat{T}_{bc;s \rightarrow u}. \quad (74)$$

These are interface identities, not revisions of SRSD. A state ledger, transition ledger, and dynamical recurrence remain different records.

11 Typed synthesis of Papers I–VI

The six preceding source works are connected, but they are not six restatements of one theorem. Their rigorous connection is a network of typed identities, specializations, and transport maps. Paper VII adds a geometric scale carrier and records those interfaces without merging the underlying objects.

11.1 Continuous contraction and discrete replay: Papers I–II

The shared exact parameter is $\lambda = \phi/(\phi + \pi)$. Define the affine map

$$C_\lambda(x) = 1 + \lambda(x - 1), \quad C_\lambda : \mathbb{R} \longrightarrow \mathbb{R}. \quad (75)$$

Then

$$C_\lambda(x) - 1 = \lambda(x - 1), \quad |C_\lambda(x) - C_\lambda(y)| = \lambda|x - y|. \quad (76)$$

Thus C_λ is a contraction with unique fixed point 1. This is a dynamical law from the contraction branch, not a consequence of the homothety action in Section 8 [1].

Paper I also treats the floor-quantized recurrence. To keep its arbitrary quantizer spacing distinct from the fixed Paper V symbol $\tau = 1/\Omega_0$, write that spacing as $\delta > 0$ and define

$$\mathbf{Q}_\delta(y) = \delta \left\lfloor \frac{y}{\delta} \right\rfloor, \quad z_{k+1} = \mathbf{Q}_\delta(\lambda z_k). \quad (77)$$

On the lattice $z = \delta d$, rescaling gives the exact identity

$$\delta^{-1} \mathbf{Q}_\delta(\lambda \delta d) = \lfloor \lambda d \rfloor. \quad (78)$$

Thus Paper I's quantized real recurrence restricts, in integer coordinates, to the exact- λ floor map $d \mapsto \lfloor \lambda d \rfloor$. This is a coordinate restriction of the quantized system, not an identification with the continuous affine map.

For the rational finite-state branch, let

$$Q = 339949771344778, \quad S = 10^{15}, \quad q = \frac{Q}{S}, \quad (79)$$

and define

$$F(d) = \left\lfloor \frac{Qd}{S} \right\rfloor, \quad F : \mathbb{Z}_{\geq 0} \longrightarrow \mathbb{Z}_{\geq 0}, \quad (80)$$

$$\rho_Q(d) = Qd - SF(d), \quad 0 \leq \rho_Q(d) < S. \quad (81)$$

The Euclidean division identity

$$Qd = SF(d) + \rho_Q(d) \quad (82)$$

makes the augmented step

$$\mathcal{L}_Q(d) = (F(d), \rho_Q(d)) \quad (83)$$

exactly reversible on its image:

$$d = \frac{SF(d) + \rho_Q(d)}{Q}. \quad (84)$$

For an orbit $d_{k+1} = F(d_k)$, put $\rho_k = Qd_k - Sd_{k+1}$. The terminal state and ordered ledger

$$\mathcal{R}_N = (d_N; \rho_0, \dots, \rho_{N-1}) \quad (85)$$

recover the entire orbit by

$$d_k = \frac{Sd_{k+1} + \rho_k}{Q}, \quad k = N - 1, \dots, 0. \quad (86)$$

This is the exact replay mechanism of Paper II [2].

The recursive reconstruction also yields the closed identity

$$Q^N d_0 = S^N d_N + \sum_{k=0}^{N-1} \rho_k S^k Q^{N-1-k}. \quad (87)$$

The same ordered record determines Paper II's maximal orbit-preserving parameter interval. Assume that the nonterminal states $d_0, \dots, d_{N-1}$ are positive, and set

$$\eta_- = \min_{0 \leq k < N} \frac{\rho_k}{S d_k}, \quad \eta_+ = \min_{0 \leq k < N} \frac{S - \rho_k}{S d_k}. \quad (88)$$

Then

$$I(\mathcal{R}_N) = [q - \eta_-, q + \eta_+) \cap (0, 1) \quad (89)$$

is exactly the maximal interval containing q for which

$$\lfloor \alpha d_k \rfloor = d_{k+1} \quad (0 \leq k < N) \quad (90)$$

holds for every $\alpha \in I(\mathcal{R}_N)$ [2]. A zero state imposes no additional restriction because it remains zero; for a ledger with trailing zeros, the displayed minima are applied to its positive prefix. Although the raw remainders depend on the chosen integer presentation (Q, S) , the normalized margins and interval are unchanged by a common rescaling of Q and S .

On the certified 15-bit domain $\mathcal{D}_{15} = \{0, 1, \dots, 2^{15} - 1\}$,

$$\lfloor \lambda d \rfloor = F(d) \quad (d \in \mathcal{D}_{15}), \quad (91)$$

although $q \neq \lambda$ [2, 5]. Combining Equations (78) and (91) gives

$$\delta^{-1} Q_\delta(\lambda \delta d) = F(d) \quad (d \in \mathcal{D}_{15}), \quad (92)$$

so the lattice coordinates of Paper I and the rational shadow of Paper II have identical forward trajectories from this domain. Since $F(d) \leq d$, those trajectories remain in $\mathcal{D}_{15}$. Moreover,

$$0 \leq F^n(d) \leq q^n d, \quad Q^{10}(2^{15} - 1) < S^{10}, \quad (93)$$

so every state in $\mathcal{D}_{15}$ reaches zero within ten steps. The witness

$$F^9(32767) = 1, \quad F^{10}(32767) = 0 \quad (94)$$

shows that the maximum extinction time is exactly ten. Hence $N = 23$ is a valid retained protocol choice for the ledger horizon, not the minimal extinction bound and not a requirement of Paper II's general theorem.

Equations (75), (80), and the geometric action T_c have different domains, codomains, and meanings. Finite floor agreement compares two scalar parameters on a declared domain; it does not identify the continuous contraction, rational floor map, or geometric homothety.

11.2 The Paper II step inside frozen SRSD: Paper III

The scalar SRSD encoder in Equation (66) gives an exact retrospective interface. Let

$$m_Q : \mathbb{Z}_{\geq 0} \longrightarrow Q\mathbb{Z}_{\geq 0}, \quad m_Q(d) = Qd. \quad (95)$$

Apply the frozen encoder at resolution $s = S$ to $m_Q(d)$. Then

$$E_S(Qd) = \left(\left\lfloor \frac{Qd}{S} \right\rfloor, Qd - S \left\lfloor \frac{Qd}{S} \right\rfloor \right) = (F(d), \rho_Q(d)). \quad (96)$$

The frozen decoder returns

$$D_S(F(d), \rho_Q(d)) = SF(d) + \rho_Q(d) = Qd, \quad (97)$$

after which division by the fixed nonzero integer Q recovers d . Along an orbit,

$$E_S(Qd_k) = (d_{k+1}, \rho_k). \quad (98)$$

Equivalently,

$$\mathcal{L}_Q = E_S \circ m_Q. \quad (99)$$

Thus Paper II's augmented transition is the pullback, along the bijection m_Q onto its image, of the already frozen SRSD coordinate map restricted to $Q\mathbb{Z}_{\geq 0}$ [3]. This adds no SRSD primitive and does not make the floor recurrence part of SRSD.

11.3 Exact arithmetic, sampling, and cubature: Papers IV–VI

Paper V supplies the exact arithmetic coordinates shared by the sequence. In addition to Equation (2),

$$\sqrt{5} = \frac{2\lambda}{\tau} - 1. \quad (100)$$

Consequently, for $a, b \in \mathbb{Q}$, the exact embedding is

$$\begin{aligned} \iota_\Omega : \mathbb{Q}(\sqrt{5}) &\hookrightarrow \mathbb{Q}(\tau, \lambda), \\ \iota_\Omega(a + b\sqrt{5}) &= (a - b) + \frac{2b}{\tau}\lambda. \end{aligned} \quad (101)$$

This is a change of exact coordinates, not a numerical approximation and not a claim that $\phi + \pi$ causes the cubature identities [5].

Let $X_f \subset S^2$ denote a radial Class-I icosahedral node set. Paper IV studies its equal-weight probability measure

$$\mu_f = \frac{1}{|X_f|} \sum_{x \in X_f} \delta_x \quad (102)$$

and detects its first invariant-harmonic obstruction [4]. For $1 \leq f \leq 9$, Paper VI keeps the same nodes and supports a strictly positive H_3 -orbit-constant probability measure

$$\nu_f = \sum_{x \in X_f} w_f(x) \delta_x, \quad w_f(x) > 0, \quad \sum_{x \in X_f} w_f(x) = 1, \quad (103)$$

with weights constant on H_3 -orbits and chosen to satisfy exact moment equations through degree

$$t_f = \begin{cases} 3f + 2, & f \text{ odd}, \\ 3f + 3, & f \text{ even}. \end{cases} \quad (104)$$

These are the certified degrees in Paper VI [6]. The required moment solution is unique for $1 \leq f \leq 8$. At $f = 9$, the moment equations have a one-dimensional affine solution space, and Paper VI selects one exact positive member by $W_{333} = 1/27$. At $f = 1$, the single orbit forces $\nu_1 = \mu_1$; from $f = 2$ onward the certified rules use nontrivial positive orbit reweighting. Thus the node set is shared while the measures are generally distinct. Equation (101) transports the exact $\mathbb{Q}(\sqrt{5})$ -coefficients of the cubature branch into the arithmetic coordinates of Paper V without changing their values.

11.4 Homothetic transport of the spherical branches

The positive scaling action also gives a precise interface from Papers IV and VI to the scale family of Paper VII. Let $h_a : S^2 \rightarrow S_a^2$ be $h_a(x) = ax$, and let the normalized surface probability measure on the radius- a sphere be

$$\bar{\sigma}_a = (h_a)_* \bar{\sigma}_1. \quad (105)$$

If the weights $w_f(x)$ integrate all polynomial restrictions of degree at most t on S^2 , then for every polynomial p of degree at most t ,

$$\int_{S_a^2} p(y) d\bar{\sigma}_a(y) = \sum_{x \in X_f} w_f(x) p(ax). \quad (106)$$

Thus exactness degree and positivity survive positive radial scaling. The same pushforward transports the equal-weight sampling measure and its harmonic error. This bridge uses the standard homothety law; the one-arcsecond planar seed neither proves nor selects the icosahedral rules.

Table 3: Exact connection map for the published source works I–VI.

Connection	Exact bridge	Retained boundary
I $\leftrightarrow$ V	Paper I uses $\lambda = \phi/(\phi + \pi)$; Paper V gives its exact field coordinates	The constant does not select the dynamics.
I $\rightarrow$ II	$\delta^{-1}Q_\delta(\lambda\delta d) = F(d)$ on $\mathcal{D}_{15}$; the ledger reconstructs and fixes the stability interval	The bridge is domain-limited; F alone is not reversible.
II $\rightarrow$ III	$\mathcal{L}_Q = E_S \circ m_Q$	This is an SRSD application, not an SRSD revision.
IV $\rightarrow$ VI	Same X_f ; $\nu_1 = \mu_1$, while $2 \leq f \leq 9$ uses positive orbit reweighting	Equal support does not imply equal weights.
V $\rightarrow$ VI	$\sqrt{5} = 2\lambda/\tau - 1$ gives exact field interoperability	Cubature validity does not depend on OMEGA naming.
IV/VI $\rightarrow$ VII	$x \mapsto ax$ pushes sampling and cubature to radius a	Homothety transports the rule; it does not generate it.

These edges connect the six published source works to the present geometric draft while preserving their separate theorem statements. The contraction/remainder kernel is concentrated in Papers I–II; frozen SRSD supplies an external exact coordinate interface; Papers IV and VI form the spherical sampling/cubature branch; and Paper V supplies the exact arithmetic interoperability used across the sequence.

12 Separation from the OMEGA drift experiment

An earlier unpublished companion computation considered whether a particular four-dimensional counter-rotation built from OMEGA ratios was preferentially selected on dynamics not generated from that frame. Its recorded result was negative under its own test protocol. That exploratory report is not used as evidence for any theorem in this paper and should be cited separately if it is placed in a stable archive.

Regardless of the outcome of that experiment, the logical separation is

$$\boxed{\text{marked geometric definition}} \neq \boxed{\text{universal dynamical preference}}. \quad (107)$$

A reference model can prevent cumulative numerical drift when the model is correct; that mechanism is generic. This paper makes no claim that its constants define a universal anchor for arbitrary dynamics.

13 Exact verification protocol

The local construction admits a short independent audit. A verifier should separate symbolic proof, interval-certified numerical evaluation, and implementation regression tests.

1. Set $\varepsilon = \pi/648000$ and retain the coordinate expressions in Equation (8) symbolically.
2. Verify

$$\|d\| = \|b\| = 1, \quad d \cdot b = \cos \varepsilon, \quad \det[d \ b] = \sin \varepsilon > 0.$$
3. Verify the chord and sagitta formulas in Equation (9). For floating-point evaluation, compute $h = 2 \sin^2(\varepsilon/4)$, not $1 - \cos(\varepsilon/2)$.
4. Construct the extension marks from Equations (13)–(14) and verify Equations (15)–(17).
5. Prove the degree laws symbolically. Use fixed rational test values of $a, c > 0$ only as deterministic implementation regressions.
6. For a finite normalized configuration X , verify $R(a) = aX$, $\text{Gr}(a) = a^2 X^T X$, and Equation (30). Confirm that $\widehat{\text{Gr}}$ and $\widehat{D}$ are unchanged under at least two distinct positive scales.
7. For every claimed dimensional embedding, verify the ambient-metric pullback in Equation (52), equality of every common named mark, and commutation with homothety in Equation (54). Do not substitute a projection for an embedding.
8. For any claimed reconstructed point or outer completion, compute the admissible set in Equation (61). Use Jacobian rank only as a local-rigidity certificate, and separately exclude disconnected, reflected, and other discrete branches before claiming global uniqueness.
9. Test Equations (73)–(74) independently of the geometry implementation.
10. Verify the finite bridge in Equation (91) over all 2^{15} inputs, check $Q^{10}(2^{15} - 1) < S^{10}$ using integer arithmetic, and reproduce the witness in Equation (94).
11. Verify directly that $E_S(Qd) = (F(d), \rho_Q(d))$ and that the frozen decoder returns Qd . This test must call the published SRSD maps without changing them.
12. Check Equation (100) symbolically and derive Equation (106) by pushforward of normalized surface measure; neither check requires floating-point arithmetic.
13. Reject any requested outer feature lacking a corresponding typed rule in $\mathcal{R}_{\text{out}}$.

Numerical comparisons should use directed intervals or a printed tolerance; floating-point equality is not a proof of an exact identity.

14 Nonclaims

The definitions and theorems above do not establish:

- that nature selects the OMEGA proportions;
- that arbitrary systems evolve by OMEGA scaling;
- that λ is an optimal computational or physical scale;

- that the one-arcsecond choice follows from ϕ or π ;
- that the golden extensions follow from the angle alone;
- that all six preceding source works are consequences of the one-arcsecond geometry or are instances of one common theorem;
- that $q = Q/S$ equals λ , rather than agreeing with it only through the certified finite floor bridge;
- that the floor recurrence is a primitive of frozen SRSD;
- that the equal-weight measure μ_f and positive cubature measure ν_f are identical throughout the family—they coincide at $f = 1$ but are reweighted from $f = 2$ onward—or that the planar seed generates either measure;
- that the unfinished outer notebook figure is unique;
- that normalized center-relative data can reconstruct a mark whose normalized vector or defining constraints have not been supplied;
- that arbitrary two-dimensional and four-dimensional observations automatically determine a three-dimensional realization;
- that an embedding, affine section, and projection are interchangeable, or that lost projection coordinates are recovered without additional data;
- that a configuration Gram matrix alone selects absolute placement, ambient orientation, or a reflection branch;
- that a fourth Euclidean coordinate is automatically time, energy, spacetime, or any other physical observable;
- that noisy, incomplete, or interval-valued measurements produce falsely exact reconstructed coordinates;
- that a marked interval and planar configuration are the same object;
- that every higher-dimensional geometry carries a canonical OMEGA ruler;
- that homotheties, group actions, torsors, logarithmic coordinates, or Euclidean scaling laws are new operations;
- that the rejected universal-anchor hypothesis is restored here;
- any quantum, cosmological, biological, or theological mechanism.

The project-specific contribution is the particular marked construction and its explicit rule separation. The surrounding mathematical language is standard.

15 Conclusion

The OMEGA marked reference geometry has a rigorous local core. A unit circle, an oriented one-arcsecond separation, and a bisecting unit direction force the endpoint coordinates, distinguished minor arc, chord, and sagitta. Declared outward extensions of lengths 1 and ϕ force bilateral totals of ϕ^2 . Together these data define a marked local kernel unique up to orientation-preserving Euclidean isometry.

Positive scale copies form a free and transitive $(\mathbb{R}_{>0}, \times)$ -family. The marked measurement spine records the complementary subinterval proportions λ, μ , while frozen SRSD may encode the scale observable through an external, exactly invertible coordinate transport. The typed pair $\mathfrak{G}_\Omega(a)$ packages the planar kernel and spine under a common diagonal action without conflating them.

For every already specified finite marked configuration, the distinguished center α , scale a , normalized vectors Ξ , and sufficient frame data reconstruct all defined marks by $p_i(a) = \alpha + a\xi_i$. Distances have degree one, configuration Gram data have degree two, and their normalized relational signatures have degree zero. Compatible isometric embeddings preserve those signatures and named subobjects across dimensions. Lower- and higher-dimensional constraints determine an intermediate realization only when their common admissible set is proved to be one point, or one orbit under the declared equivalence group. This reconstruction criterion does not fill in absent notebook rules.

The preceding source works connect through explicit interfaces rather than a single retrospective theory: Paper V supplies the shared exact constants, Papers I and II supply the contraction and reversible remainder ledger, Paper III supplies the frozen external coordinate representation, and Papers IV and VI supply the equal-weight and positive-weight spherical branches on the same nodes, coinciding at $f = 1$ and reweighted thereafter. The identities in Section 11 make every connection independently checkable while retaining each paper's separate claims.

The local kernel and center-relative transport law are proved; the full outer completion remains to be specified:

OMEGA supplies a declared marked similarity type; center, normalized relations, frame, and scale reconstruct every already-defined mark; positive real numbers supply the scaling action; and separate constraints or laws must supply any missing geometry or dynamics.	(108)
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A Numerical audit values

The following values are implementation checks only:

$$\varepsilon \approx 4.848136811095360 \times 10^{-6}, \quad (109)$$

$$\sin(\varepsilon/2) \approx 2.424068405545306 \times 10^{-6}, \quad (110)$$

$$2 \sin(\varepsilon/2) \approx 4.848136811090612 \times 10^{-6}, \quad (111)$$

$$1 - \cos(\varepsilon/2) = 2 \sin^2(\varepsilon/4) \approx 2.938053817385797 \times 10^{-12}, \quad (112)$$

$$\phi \approx 1.618033988749895, \quad (113)$$

$$\phi^2 \approx 2.618033988749895, \quad (114)$$

$$\Omega_0 \approx 4.759626642339688, \quad (115)$$

$$\tau \approx 0.210100513158828, \quad (116)$$

$$\lambda \approx 0.339949771344779, \quad (117)$$

$$\mu \approx 0.660050228655221. \quad (118)$$

The exact formulas, rather than the rounded values, govern proofs and reference implementations.

B Constraint ledger for a future outer completion

A future revision may turn the outer diagram into a second rigidity theorem by publishing a machine-readable constraint ledger $\mathcal{R}_{\text{out}}$. Each feature must receive one of four logical statuses:

DEFINED	introduced explicitly as part of the construction;
DERIVED	forced by previously declared rules and accompanied by a derivation;
CONJECTURAL	exactly specified but not yet proved to exist or be unique as claimed;
INTERPRETIVE	a drawing or semantic label without sufficient mathematical constraints.

At minimum, every added mark or curve should record:

- unique identifier, label, geometric type, ambient dimension, and dimensional memberships;
- parent objects, distinguished center, exact defining equations, and normalized center-relative coordinates when known;
- distance, angle, incidence, tangency, line, plane, circle, sphere, and subspace constraints that are intended to hold;
- orientation signs, intersection order, and branch choices for every multivalued construction;
- scale degree of every dimensionful field and the associated scale-normalized relation;
- logical status, derivation, source drawing, and verification method;
- a symbolic or interval-certified existence witness; and
- a local-rigidity certificate and a separate global-uniqueness status.

A portable schema should therefore contain fields equivalent to

<code>id</code>	<code>label</code>	<code>type</code>
<code>ambient_dimension</code>	<code>parent_objects</code>	<code>reference_center</code>
<code>defining_equations</code>	<code>distance_constraints</code>	<code>angle_constraints</code>
<code>incidence_constraints</code>	<code>tangency_constraints</code>	<code>orientation</code>
<code>branch_choice</code>	<code>scale_degree</code>	<code>normalized_coordinates</code>
<code>status</code>	<code>existence_witness</code>	<code>local_rigidity</code>
<code>global_uniqueness</code>	<code>derivation</code>	<code>source_drawing</code>
<code>verification_method</code>		

The diamond labeled “Space” may be promoted to a marked point s only after that type and role are declared. At that stage its basic center-relative fields would be

$$r_s = s - \alpha, \quad \xi_s = \frac{s - \alpha}{a}, \quad (119)$$

together with exact relations to d, b , its plane or subspace membership, its intended relations to Echo and Loco, and its existence and uniqueness certificates. The same requirement applies to proposed points e and ℓ for Echo and Loco:

$$r_e = e - \alpha, \quad r_\ell = \ell - \alpha. \quad (120)$$

Their drawn placement alone supplies none of these coordinates.

For every applicable pair of marks, a relational ledger may retain

$$D_{ij} = \|p_i - p_j\|, \quad S_{ij} = D_{ij}^2, \quad K_{ij} = (p_i - \alpha) \cdot (p_j - \alpha), \quad \theta_{ij}, \quad (121)$$

along with incidence flags, orientation signs, common-subspace membership, and the normalized quantities D_{ij}/a and K_{ij}/a^2 . These fields make the scaling behavior and the distinction between relational and absolute orientation data auditable.

The intended upgrade path is

$$\text{INTERPRETIVE} \longrightarrow \text{DEFINED} \longrightarrow \text{DERIVED}, \quad (122)$$

but only where recovered or newly declared rules justify each transition. Otherwise the feature remains interpretive or conjectural. Such a ledger would convert the visual Echo–Loco completion into an exact constraint system while leaving the proved local theorem unchanged.

Availability and acknowledgements

The mathematical statements require no empirical dataset. A cryptographic digest certifies byte identity, not mathematical truth. Any executable verifier or companion experiment distributed with this paper should be versioned, hashed, and cited independently of the theorem text. The author does not claim that a public Paper VII verifier or certificate package exists at this prepublication stage. The author acknowledges the value of independent controls in separating a valid reference-frame mechanism from an unsupported universal-anchor claim. The notebook drawings are constructional provenance; the exact definitions printed here govern every mathematical claim.

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