
OMEGA Marked Reference Geometry

A One-Arcsecond Seed, Center-Relative Reconstruction,
and Positive-Homothety Scaling

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Paper VII in the OMEGA / OMEGA Naught sequence. This paper formalizes the local marked Euclidean kernel, center-relative scale transport, and the exact conditions under which compatible dimensional constraints reconstruct a marked realization.

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Abstract

This paper defines a marked Euclidean reference geometry from an oriented unit circle, a distinguished center, a unit bisecting direction, and an ordered pair of radius endpoints separated by exactly one arcsecond. The minor arc, chord, and sagitta are forced. Declared outward collinear extensions of lengths 1 and ϕ then determine the local marked kernel, which is unique up to orientation-preserving Euclidean isometry.

The local construction is extended by a center-relative realization theorem. An exactly specified normalized configuration $\Xi = (\xi_1, \dots, \xi_N)$, center α , and scale $a > 0$ reconstruct every defined mark by $p_i(a) = \alpha + a\xi_i$. Configuration Gram data scale by a^2 , while normalized Gram matrices, normalized distances, angles, and incidence relations are scale-invariant. Compatible isometric embeddings preserve these data across dimensions. Lower- and higher-dimensional constraints reconstruct an intermediate realization precisely when their common admissible set is proved unique, including orientation and discrete branch choices. The larger notebook completion remains a constraint problem: drawn marks become mathematical objects only after their defining relations are stated.

Keywords. marked Euclidean geometry; arcsecond; golden ratio; homothety; center-relative coordinates; Gram matrix; compatible embedding; constraint reconstruction; local rigidity; similarity invariant

1 Construction and scope

The paper has one geometric spine. First, the one-arcsecond seed and declared golden extensions determine a rigid marked planar kernel. Second, normalized vectors from the distinguished center transport that kernel, and any later exactly specified marks, through positive scale. Third, compatible dimensional embeddings and constraint intersections describe when the same relational data determine a realization in a larger or intermediate ambient space.

Claim boundary 1.1 (Principal statement) *The local OMEGA reference kernel is uniquely determined, up to orientation-preserving Euclidean isometry, by its declared rule set. The center-relative theorem transports every already-defined mark exactly. Additional two-, three-, or four-dimensional marks are determined only when their declared constraint system has a unique admissible realization.*

2 Constants, units, and conventions

2.1 Golden ratio

Let

$$\phi = \frac{1 + \sqrt{5}}{2}, \quad \phi^2 = \phi + 1. \quad (1)$$

Only the identity $\phi^2 = 1 + \phi$ is used in the bilateral extension rule.

2.2 One arcsecond

The angular seed is

$$\varepsilon = 1'' = \frac{1}{3600}^\circ = \frac{\pi}{648000} \text{ radians}. \quad (2)$$

On a circle of radius a , the corresponding minor-arc length is $a\varepsilon$. The angle is dimensionless; the arc length depends on the chosen radius.

2.3 Orientation and marked data

We work in an oriented Euclidean plane. A marked object includes its distinguished points, curves, directions, labels, and roles. An isometry of a marked object must preserve this typed data and the endpoint ordering, not merely the unlabelled point set. For nonzero vectors v, w , write $\text{ang}^+(v, w) \in [0, 2\pi)$ for the positively oriented angle from v to w , and write R_θ for positively oriented rotation through θ .

3 The one-arcsecond Euclidean seed

Definition 3.1 (Oriented one-arcsecond seed) *An oriented one-arcsecond seed is the marked tuple*

$$\mathcal{S}_\varepsilon = (E^2, \text{or}, \alpha, C, u; d, b; A_\varepsilon),$$

where E^2 is an oriented Euclidean plane, $\alpha \in E^2$, $C = \{x : \|x - \alpha\| = 1\}$, and u is a unit direction based at α . Its ordered endpoints and distinguished minor arc are

$$d = \alpha + R_{-\varepsilon/2}u, \quad b = \alpha + R_{\varepsilon/2}u, \quad (3)$$

$$A_\varepsilon = \{\alpha + R_\theta u : -\varepsilon/2 \leq \theta \leq \varepsilon/2\}. \quad (4)$$

Thus $\text{ang}^+(d - \alpha, b - \alpha) = \varepsilon$, and u is the internal bisector of the distinguished minor arc.

The labels d and b record left and right endpoints in the canonical placement below. Exchanging them changes the oriented marked object. If an open-circle rendering is desired while retaining the endpoint marks, define the endpoint-free distinguished arc and its complement by

$$A_\varepsilon^\circ = \{\alpha + R_\theta u : -\varepsilon/2 < \theta < \varepsilon/2\}, \quad C^\circ = C \setminus A_\varepsilon^\circ. \quad (5)$$

The kernel itself records A_ε as a distinguished arc; it does not silently delete it.

Proposition 3.2 (Canonical coordinates) *Every oriented one-arcsecond seed has an orientation-preserving Euclidean placement with*

$$\alpha = (0, 0), \quad u = (0, -1), \quad (6)$$

and

$$d = \left(-\sin \frac{\varepsilon}{2}, -\cos \frac{\varepsilon}{2}\right), \quad b = \left(\sin \frac{\varepsilon}{2}, -\cos \frac{\varepsilon}{2}\right). \quad (7)$$

The chord $[d, b]$ is horizontal.

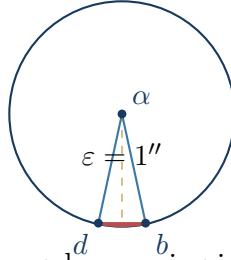
Proof. Translate α to the origin and rotate u to the negative y -axis. Equation (3) then gives the displayed coordinates. The two y -coordinates agree. $\square$

Proposition 3.3 (Forced local measurements) *For the unit-radius seed, the distinguished minor-arc length, chord length, and sagitta are*

$$L_{\text{arc}} = \varepsilon, \quad L_{\text{chord}} = 2 \sin \frac{\varepsilon}{2}, \quad h = 1 - \cos \frac{\varepsilon}{2} = 2 \sin^2 \frac{\varepsilon}{4}. \quad (8)$$

The complementary circular arc has length $2\pi - \varepsilon$.

Proof. The unit-circle arc-length formula gives $L_{\text{arc}} = \varepsilon$. Bisecting the isosceles triangle subtended by the chord gives half-chord $\sin(\varepsilon/2)$ and apothem $\cos(\varepsilon/2)$. The stable sagitta identity is $1 - \cos x = 2 \sin^2(x/2)$. $\square$



Schematic only: the angular opening is greatly exaggerated.
Exact chord length: $2 \sin(\varepsilon/2)$.

Figure 1: The exact local seed. The red curve is the distinguished minor arc A_ε ; an open-circle rendering may remove A_ε° .

The exact chord length is not equal to the radian measure ε , although the two are extremely close:

$$2 \sin \frac{\varepsilon}{2} = \varepsilon - \frac{\varepsilon^3}{24} + O(\varepsilon^5). \quad (9)$$

Thus “one arcsecond apart horizontally” means that the endpoints have one arcsecond of central angular separation and their connecting chord is horizontal in the canonical placement.

Translating the chord line to $y = 0$ gives

$$d = \left(-\sin \frac{\varepsilon}{2}, 0\right), \quad b = \left(\sin \frac{\varepsilon}{2}, 0\right), \quad \alpha = \left(0, \cos \frac{\varepsilon}{2}\right). \quad (10)$$

4 Golden-ratio extensions

Let

$$e = \frac{b - d}{\|b - d\|} \quad (11)$$

be the positively directed unit vector along the chord. In Equation (10), $e = (1, 0)$.

Definition 4.1 (Declared bilateral extensions) *Define*

$$d_1 = d - e, \quad d_2 = d - \phi^2 e, \quad (12)$$

$$b_1 = b + e, \quad b_2 = b + \phi^2 e. \quad (13)$$

The directions are outward from the seed chord, and the points occur in the order d_2, d_1, d, b, b_1, b_2 along the oriented baseline.

Proposition 4.2 (Golden decomposition) *The bilateral extensions satisfy*

$$\|d - d_1\| = \|b - b_1\| = 1, \quad (14)$$

$$\|d_1 - d_2\| = \|b_1 - b_2\| = \phi, \quad (15)$$

$$\|d - d_2\| = \|b - b_2\| = \phi^2. \quad (16)$$

Proof. The first and third equalities follow from $\|e\| = 1$. For the intervening segment,

$$\|d_1 - d_2\| = \|(-e) - (-\phi^2 e)\| = \phi^2 - 1 = \phi,$$

and the right-hand calculation is identical. $\square$

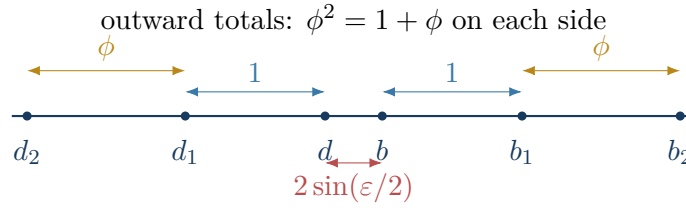


Figure 2: The declared collinear golden extensions. Horizontal proportions are schematic; the central chord is too small to resolve at this scale.

The golden extensions are not consequences of the one-arcsecond angle. They are additional declared rules. Once declared, their positions are forced.

5 The local OMEGA marked kernel

Definition 5.1 (Local OMEGA reference kernel) *The unit-scale local OMEGA reference kernel is*

$$\mathcal{K}_\Omega(1) = (E^2, \text{or}, \alpha, C, u; d, b; A_\varepsilon; d_1, d_2, b_1, b_2), \quad (17)$$

subject to the definitions above.

Theorem 5.2 (Local rigidity) *Any two realizations of $\mathcal{K}_\Omega(1)$ are related by a unique orientation-preserving Euclidean isometry preserving every named mark and its type. Equivalently, the local OMEGA reference kernel is uniquely determined up to orientation-preserving Euclidean isometry.*

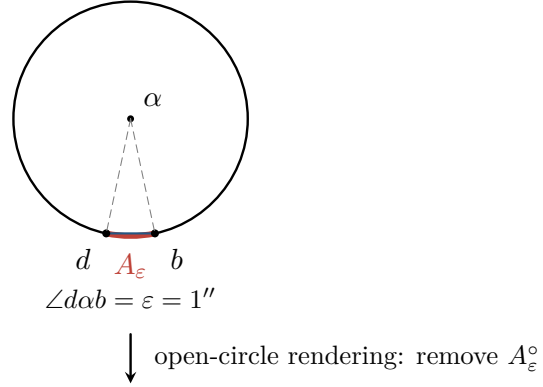
Proof. Given two realizations, there is a unique direct Euclidean isometry sending α to α' and u to u' : translate the center and apply the unique orientation-preserving rotation matching the unit directions. Equation (3) forces the images of d, b , and Equation (4) forces the image of A_ε . The directed chord vector e is therefore sent to e' , so Equations (12)–(13) force the images of all extension marks. If two direct isometries had these properties, their quotient would fix a point and a nonzero directed vector and hence be the identity. $\square$

Corollary 5.3 (Predetermined local shape) *Once the endpoint ordering, unit radius, bisecting unit direction, plane orientation, and golden-extension rules are fixed, every distance and incidence relation among the marks of $\mathcal{K}_\Omega(1)$ is predetermined.*

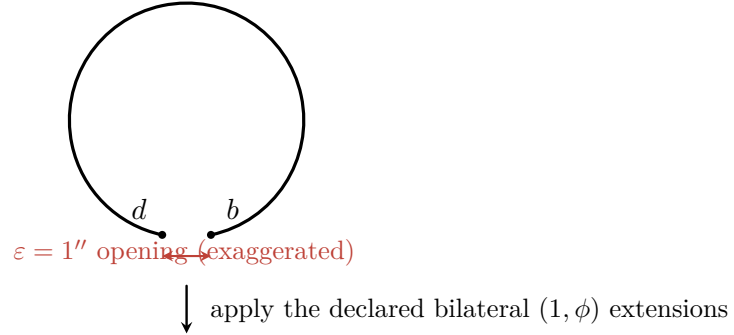
Formation of the OMEGA shape

The local kernel can be read directly as a construction sequence. The opening below is schematic: the true central separation is exactly $\varepsilon = 1''$ and is far too small to resolve at page scale.

(i) Oriented one-arcsecond seed



(ii) Endpoint-free circular body



(iii) OMEGA rendering of the marked kernel

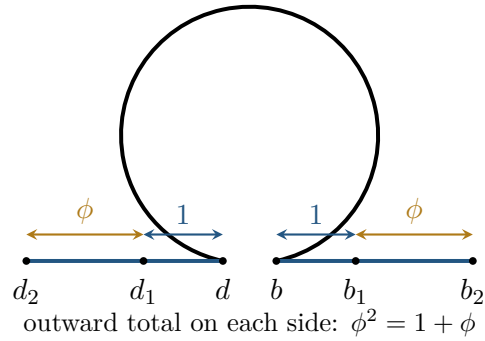


Figure 3: Formation of the OMEGA shape from the local marked kernel. The unit circle first fixes the ordered one-arcsecond seed. Using the endpoint-free rendering $C^\circ = C \setminus A_\varepsilon^\circ$ opens the circle at the distinguished minor arc. The declared outward collinear extension marks then supply the two feet, with successive lengths 1 and ϕ on each side. The angular opening is greatly exaggerated; the exact construction data are unchanged.

The precise motivating statement is therefore:

The one-arcsecond angle fixes the circular seed up to similarity; unit radius fixes its direct-isometry class. The additional bilateral $(1, \phi)$ extension rule fixes the local marked kernel.

No unlisted outer arc, box edge, or named outer point follows from the angle alone.

6 Outer-completion data

The notebook's larger arcs, frame, and named marks are the next constraint layer. A drawing can motivate their construction, but visual placement is not an incidence or metric axiom.

Let $\mathcal{R}_{\text{out}}$ denote their typed points and curves, defining equations, dimensional memberships, orientation data, and branch choices, with $\mathcal{K}_{\Omega}(1)$ retained as a marked subobject. An outer rigidity theorem requires a realization, uniqueness up to the declared equivalence group, and explicit treatment of mirror and other discrete branches. Section 9 gives the record required for Space, Echo, Loco, and the remaining outer features.

7 Positive-homothety scaling

7.1 Center-relative normalized configurations

Let $\alpha \in \mathbb{R}^n$ be a distinguished center. For each marked point p_i , its primary center-relative vector is

$$r_i = p_i - \alpha. \quad (18)$$

If $a > 0$ is the common scale, define the normalized vector

$$\xi_i = \frac{p_i - \alpha}{a}. \quad (19)$$

Thus absolute coordinates are recovered from center, scale, and normalized data by

$$p_i(a) = \alpha + a\xi_i. \quad (20)$$

Theorem 7.1 (OMEGA center-relative reconstruction principle) *Let $\Xi = (\xi_1, \dots, \xi_N) \subset \mathbb{R}^n$ be an exactly specified ordered finite marked configuration relative to the vector-space origin. For every $\alpha \in \mathbb{R}^n$ and $a > 0$, Equation (20) determines a unique ordered marked realization. For $c > 0$, the homothety about α ,*

$$H_{\alpha,c}(x) = \alpha + c(x - \alpha), \quad (21)$$

satisfies

$$p_i(ca) = H_{\alpha,c}(p_i(a)), \quad p_i(ca) - \alpha = c(p_i(a) - \alpha). \quad (22)$$

Consequently, differences, lengths, squared lengths, inner products, and Euclidean k -volumes have homogeneous degrees 1, 1, 2, 2, k , respectively. Angles, normalized inner products, scale-free ratios, and all marked incidence and orientation relations preserved by a positive similarity have degree zero.

Proof. The reconstruction formula gives each point explicitly, so uniqueness is immediate. Equation (22) follows by substitution. Every remaining statement follows from multilinearity of inner products and determinants and cancellation of common positive factors in normalized quantities. $\square$

For the complete configuration, collect the normalized and center-relative vectors as columns:

$$X = [\xi_1 \cdots \xi_N], \quad R(a) = [p_1(a) - \alpha \cdots p_N(a) - \alpha] = aX. \quad (23)$$

Its *configuration Gram matrix* is

$$\text{Gr}(a) = R(a)^T R(a) = a^2 X^T X, \quad \widehat{\text{Gr}} = \frac{1}{a^2} \text{Gr}(a) = X^T X. \quad (24)$$

Thus $\text{Gr}(ca) = c^2 \text{Gr}(a)$, while the normalized Gram matrix is scale-independent. Its entries give center distances and pairwise distances:

$$\text{Gr}_{ii}(a) = \|p_i(a) - \alpha\|^2, \quad (25)$$

$$\|p_i(a) - p_j(a)\|^2 = \text{Gr}_{ii}(a) + \text{Gr}_{jj}(a) - 2\text{Gr}_{ij}(a). \quad (26)$$

Equivalently, the normalized distance matrix

$$\widehat{D}_{ij} = \frac{\|p_i(a) - p_j(a)\|}{a} = \|\xi_i - \xi_j\| \quad (27)$$

does not depend on a .

Remark 7.2 (Metric data and oriented realization) *The normalized Gram matrix determines a center-relative Euclidean configuration only up to an orthogonal transformation on its span. It does not select an absolute ambient placement or distinguish a realization from its reflection. A specific oriented realization therefore also requires sufficient anchor or frame data; reflection and other discrete branches must not be discarded silently.*

One convenient exact record for a completely specified realization is

$$\mathfrak{D} = (\alpha, a, E, \Xi, \mathcal{C}, \mathcal{M}), \quad (28)$$

where E is an oriented ambient frame, $\mathcal{C}$ is an exact typed constraint ledger, Ξ is expressed relative to E , and $\mathcal{M}$ records mark types, labels, incidence roles, and dimensional memberships. Under T_c , only a and the dimensionful entries of $\mathcal{C}$ acquire their prescribed powers of c ; $E, \Xi, \mathcal{M}$, and the scale-free relations remain fixed. This tuple is bookkeeping for standard Euclidean data, not a new geometric primitive.

In compact form,

$$\boxed{\text{center} + \text{relations} + \text{frame} + \text{scale} \implies \text{reconstruction}}, \quad (29)$$

$$\boxed{\text{scale state changes; normalized relational geometry does not}}.$$

7.2 Scaled planar kernels

The local kernel is the special case with canonical center $\alpha = 0$. For $a > 0$, define the scaled data componentwise by

$$C_a = aC, \quad A_{\varepsilon, a} = aA_{\varepsilon}, \quad d_a = ad, \quad b_a = ab, \quad (d_{1,a}, d_{2,a}, b_{1,a}, b_{2,a}) = a(d_1, d_2, b_1, b_2), \quad (30)$$

while the marked unit direction is

$$u_a = u. \quad (31)$$

Let $\mathcal{K}_\Omega(a)$ be the resulting typed marked kernel. Its circle has radius a , its central angle remains ε , and its extension lengths are $a, a\phi, a\phi^2$.

For $c > 0$, the origin-centered homothety $H_c(x) = cx$ acts on the point and curve data and induces the unchanged normalized direction u . Define

$$T_c(\mathcal{K}_\Omega(a)) = \mathcal{K}_\Omega(ca). \quad (32)$$

Theorem 7.3 (Scaling action) *The map*

$$\rho : \mathbb{R}_{>0} \times \{\mathcal{K}_\Omega(a) : a > 0\} \longrightarrow \{\mathcal{K}_\Omega(a) : a > 0\}, \quad \rho(c, \mathcal{K}_\Omega(a)) = \mathcal{K}_\Omega(ca), \quad (33)$$

is a free and transitive action of $(\mathbb{R}_{>0}, \times)$.

Proof. The identity and composition laws are

$$T_1(\mathcal{K}_\Omega(a)) = \mathcal{K}_\Omega(a), \quad T_b(T_c(\mathcal{K}_\Omega(a))) = \mathcal{K}_\Omega(bca) = T_{bc}(\mathcal{K}_\Omega(a)).$$

Also $T_c^{-1} = T_{1/c}$. If $T_c(\mathcal{K}_\Omega(a)) = \mathcal{K}_\Omega(a)$, equality of circle radii gives $ca = a$, hence $c = 1$. Given $\mathcal{K}_\Omega(a)$ and $\mathcal{K}_\Omega(b)$, the unique $c = b/a$ sends the first to the second. $\square$

Thus the canonically positioned family $\{\mathcal{K}_\Omega(a) : a > 0\}$ is a torsor for $(\mathbb{R}_{>0}, \times)$.

7.3 Scaling laws

A scalar quantity Q is homogeneous of degree k if

$$Q(T_c X) = c^k Q(X). \quad (34)$$

Table 1: Scale-dependent quantities and invariants.

Quantity at scale a	Under $a \mapsto ca$	Degree/status
Circle radius a	multiplied by c	1
Distinguished minor-arc length $a\varepsilon$	multiplied by c	1
Chord $2a \sin(\varepsilon/2)$	multiplied by c	1
Sagitta $a(1 - \cos(\varepsilon/2))$	multiplied by c	1
Extension lengths $a, a\phi, a\phi^2$	multiplied by c	1
Squared distance and planar area	multiplied by c^2	2
Configuration Gram entries	multiplied by c^2	2
Angle ε and unit direction u	unchanged	0
Ratios $1 : \phi : \phi^2$	unchanged	0
Normalized distances and normalized Gram data	unchanged	0
Labels, order, and incidence	preserved for $c > 0$	invariant

The family is deliberately indexed by $a > 0$. At $c = 0$ the configuration collapses. In the plane, $c < 0$ gives $H_c = R_\pi \circ H_{|c|}$: it takes the object out of the chosen canonical placement. Thus the acting group is $(\mathbb{R}_{>0}, \times)$.

The action describes exact scale transport. It does not, by itself, choose a physical or computational trajectory through the scale family.

8 Compatible dimensional realizations and conditional reconstruction

Increasing ambient dimension does not make distinct objects identical and does not create missing information. Compatibility means that explicit maps preserve the metrics, named marks, and relations claimed to survive.

For a positive-definite ambient metric matrix G_n , write $\langle x, y \rangle_{G_n} = x^T G_n y$. The center-relative homothety $H_{\alpha, c}(x) = \alpha + c(x - \alpha)$ is valid in every finite Euclidean dimension: lengths scale by c , inner products by c^2 , and k -dimensional volumes by c^k . What changes with dimension is the amount of frame and constraint information required to identify a realization.

8.1 Compatible embeddings and marked subobjects

Let $(\mathbb{R}^m, G_m, \alpha_m)$ and $(\mathbb{R}^n, G_n, \alpha_n)$, with $m < n$, be centered Euclidean realizations. Let $J_m^n \in \mathbb{R}^{n \times m}$ have rank m , and define the affine embedding

$$\iota_m^n(x) = \alpha_n + J_m^n(x - \alpha_m). \quad (35)$$

It is isometric precisely when

$$(J_m^n)^T G_n J_m^n = G_m. \quad (36)$$

This is a pullback identity for the *ambient metric matrices*. If X_m contains normalized vectors for common named marks and $X_n = J_m^n X_m$, then their configuration Gram matrices agree:

$$X_n^T G_n X_n = X_m^T G_m X_m. \quad (37)$$

The embedding also commutes with center-relative scaling:

$$\iota_m^n \circ H_{\alpha_m, c} = H_{\alpha_n, c} \circ \iota_m^n. \quad (38)$$

For the canonical chain, one may take $J_n^{n+1}x = (x, 0)$. Then the image is the declared affine coordinate subspace and all common named marks are preserved. More generally, a compatible chain

$$\mathcal{X}_2 \xrightarrow{\iota_2^3} \mathcal{X}_3 \xrightarrow{\iota_3^4} \mathcal{X}_4 \quad (39)$$

requires every common named mark and relation to commute with the displayed embeddings. This is the precise sense in which a lower-dimensional realization survives as a marked subobject.

A projection is not an embedding. A map from four dimensions to three may discard a coordinate or identify distinct points. Exact recovery from such a projection requires additional data that separate every point in the relevant fiber. Likewise, a three-dimensional embedded subobject, an affine section, and a projected image are different typed constructions.

8.2 Frames generated from the seed

In an oriented standard Euclidean three-space, let

$$r_d = d - \alpha, \quad r_b = b - \alpha, \quad \|r_d\| = \|r_b\| = a, \quad \angle(r_d, r_b) = \varepsilon. \quad (40)$$

Because $0 < \varepsilon < \pi$, the vectors are noncollinear and their internal bisector and ordered unit normal are

$$u = \frac{r_d/\|r_d\| + r_b/\|r_b\|}{\|r_d/\|r_d\| + r_b/\|r_b\|\|}, \quad n = \frac{r_d \times r_b}{\|r_d \times r_b\|}. \quad (41)$$

With $v = n \times u$, the ordered triple (u, v, n) , together with α , is an oriented orthonormal local frame. The order (d, b) and the ambient orientation are part of the data; reversing either can change the normal. Positive homothety leaves this normalized frame unchanged.

In $\mathbb{R}^n$, an ordered independent frame $E = (e_1, \dots, e_n)$ gives unique coordinates

$$p_i - \alpha = \sum_{k=1}^n x_{ik} e_k. \quad (42)$$

For an orthonormal frame, these are the usual inner-product coordinates. In four dimensions there is no ordinary vector-valued cross product with the role used in Equation (41); orientation is recorded by an ordered four-frame, for example through

$$\text{sgn det}[e_1 \ e_2 \ e_3 \ e_4]. \quad (43)$$

8.3 Constraint-intersection reconstruction

Suppose an unknown ordered tuple $x = (x_1, \dots, x_M) \in (\mathbb{R}^3)^M$ must obey exact constraints inherited from a declared two-dimensional marked subobject, constraints pulled back from a compatible four-dimensional reference, and intrinsic three-dimensional incidence, orientation, and branch conditions. After expressing all constraints in the same three-dimensional coordinates, write its admissible set as

$$\mathcal{A} = \{x : F_2(x) = 0, \ F_3(x) = 0, \ F_4(x) = 0, \ B(x)\}, \quad (44)$$

where B collects inequalities, orientation signs, and discrete branch choices.

Proposition 8.1 (Intermediate-dimensional reconstruction criterion) *The tuple is exactly reconstructed when $\mathcal{A}$ is a singleton. More generally, relative to a declared equivalence group $\mathcal{E}$, it is unique up to equivalence exactly when $\mathcal{A}$ is one $\mathcal{E}$ -orbit. Compatible lower- and higher-dimensional data do not imply either conclusion unless existence and this uniqueness condition are proved.*

This criterion may be summarized by the typed constraint pattern

$$\mathcal{X}_2 \longrightarrow \mathcal{X}_3 \longleftarrow \mathcal{X}_4, \quad \mathcal{A}/\mathcal{E} = \{[x_*]\}, \quad (45)$$

where the arrows denote declared constraint transport, not automatic recovery or generation of the intermediate object.

For one unknown point, typical exact equations include

$$\|x - a_i\|^2 = r_i^2, \quad (x - \alpha) \cdot v_j = c_j, \quad (x - \alpha) \cdot n = h. \quad (46)$$

Three generic sphere equations in $\mathbb{R}^3$ commonly leave zero, one, or two solutions; when two remain, they are mirror branches across the anchor plane. A further independent condition can select one branch.

For a smooth equality-constraint map $F : (\mathbb{R}^3)^M \rightarrow \mathbb{R}^r$, after the allowed Euclidean gauge freedoms are fixed, full column rank of $DF(x_*)$ is a sufficient test that x_* is locally isolated. It does not prove global uniqueness: disconnected or reflected solutions must still be excluded. Accordingly,

$$\text{existence} \neq \text{local rigidity} \neq \text{global rigidity}. \quad (47)$$

The same constraint discipline applies when a three-dimensional realization is a declared affine subspace $H \subset \mathbb{R}^4$. A point belongs to that realization only if it lies in H , and its three-dimensional coordinates require a declared affine isometry $\mathbb{R}^3 \rightarrow H$. A fourth coordinate is mathematical data, not automatically time, energy, spacetime, or another physical quantity.

Exact symbolic inputs may support exact reconstruction. Certified interval or measured approximate inputs should instead produce certified regions or uncertainty sets unless additional analysis proves an exact conclusion. The reference geometry does not turn insufficient or noisy observations into unique exact coordinates.

This section supplies compatible families and a reconstruction criterion. It does not assert that every higher-dimensional geometry carries a canonical OMEGA measurement map, or that the current notebook marks already supply the required constraint system.

9 Outer completion as a constraint problem

The local theorem does not determine the marks called *Space*, *Echo*, or *Loco*, nor the outer arcs and frame shown in the notebook drawing. Those features become mathematical objects only through an exact constraint system $\mathcal{R}_{\text{out}}$.

Each proposed feature should be assigned one of four statuses:

DEFINED	introduced by an explicit construction rule;
DERIVED	forced by prior rules and accompanied by a proof;
CONJECTURAL	exactly specified, but not yet proved to exist or be unique;
INTERPRETIVE	present in a drawing or label without sufficient constraints.

For a proposed point s , the minimum center-relative record is

$$r_s = s - \alpha, \quad \xi_s = \frac{s - \alpha}{a}, \quad (48)$$

together with its ambient dimension, defining equations, incidences, distances, angles, subspace membership, orientation, and branch choice. The same requirement applies to any proposed points e and ℓ representing Echo and Loco. Their placement in a drawing does not supply coordinates or prove uniqueness.

For every applicable pair of marks, the relational ledger may retain

$$D_{ij} = \|p_i - p_j\|, \quad S_{ij} = D_{ij}^2, \quad K_{ij} = (p_i - \alpha) \cdot (p_j - \alpha), \quad \theta_{ij}, \quad (49)$$

with incidence flags, orientation signs, common-subspace membership, and the normalized quantities D_{ij}/a and K_{ij}/a^2 . Each field must also record its scale degree and whether it is given or derived.

A completed outer constraint system must separately establish:

1. existence of an admissible realization;
2. local rigidity after the declared Euclidean gauge is fixed;
3. global uniqueness, including the exclusion or explicit retention of reflected and disconnected branches.

Only then may an outer feature be promoted from a declared construction to a derived consequence. This requirement preserves the proved local kernel while giving the notebook geometry a precise route toward completion.

10 Conclusion

The OMEGA construction has a concise rigorous core. A unit circle, an ordered one-arcsecond separation, orientation, and a bisecting unit direction determine the local seed. Declared outward extensions of lengths 1 and ϕ complete the marked baseline. The resulting local kernel is unique up to orientation-preserving Euclidean isometry.

For every exactly specified marked configuration, the data (α, a, Ξ) , together with sufficient oriented frame information, reconstruct all defined points by

$$p_i(a) = \alpha + a\xi_i.$$

Distances have homogeneous degree 1, squared distances and Gram entries have degree 2, and normalized distances, normalized Gram data, angles, incidences, and scale-free ratios have degree 0. Positive homothety changes the scale state while preserving the normalized relational geometry.

Compatible embeddings carry this information between dimensions. Constraints from a marked lower-dimensional subobject and a higher-dimensional reference configuration determine an intermediate realization only when their common admissible set is proved to contain a single realization, or a single orbit under the declared equivalence group. Dimension alone contributes no missing information.

Thus the present result is not a completed theory of the outer drawing. It is a rigidity theorem and reconstruction framework that states exactly what the remaining geometry would have to satisfy:

A distinguished center, normalized relational data, sufficient oriented frame information, and a positive scale reconstruct every already-defined mark. Any additional mark requires explicit constraints and a proof of existence and uniqueness.	(50)
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